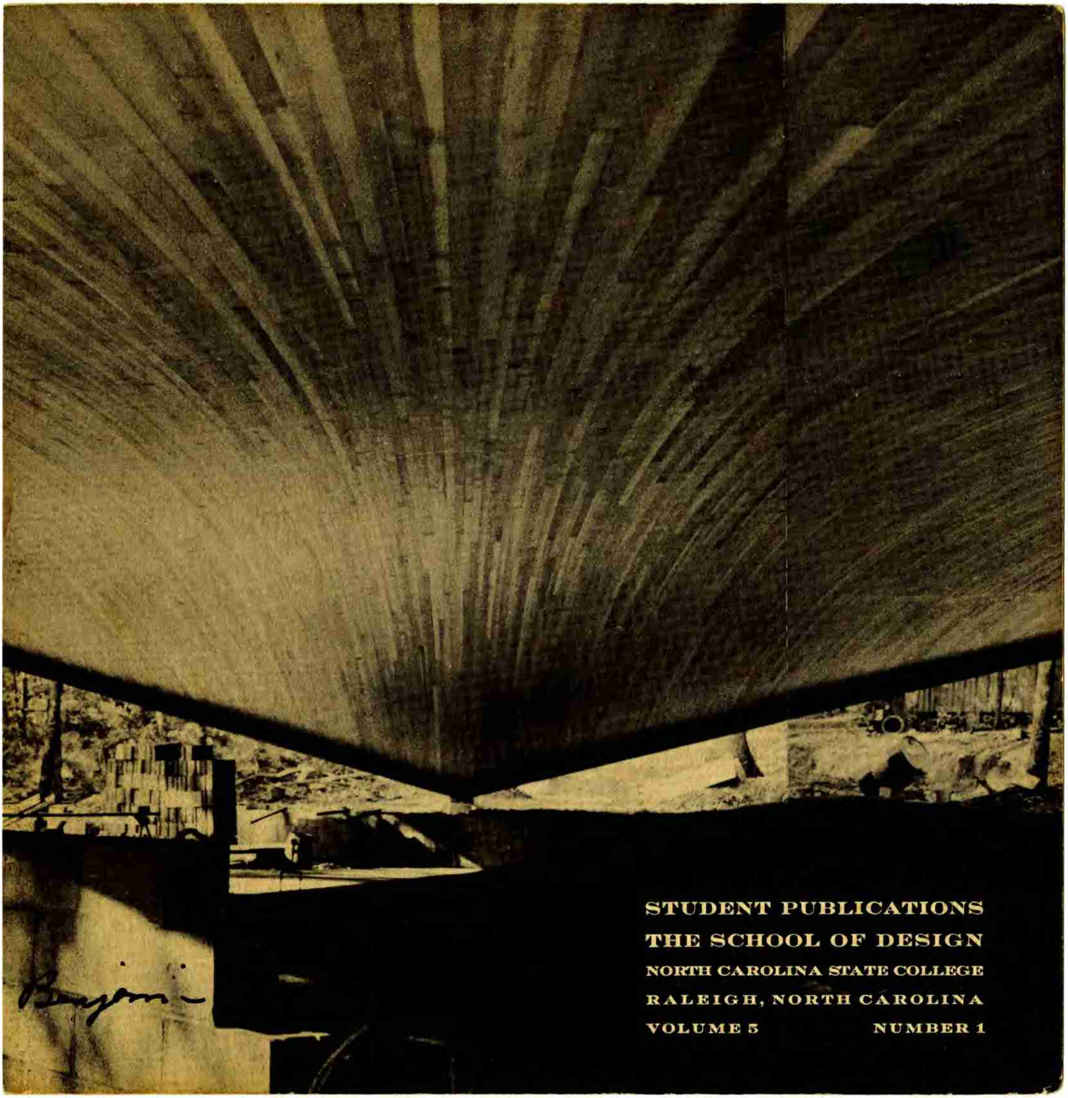

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Picture of a hyperbolic paraboloid, built with three layers of laminated timber for a span of 86′-0″. Tangential stresses compressing the steel edges are transmitted to concrete buttresses which are held by three prestressed cables of 1″ diameter for 75,000 pounds each, constant thickness 2¼″. Construction method suggested by Eng. Atilio Gallo, from Buenos Aires, Argentina. Built in Raleigh, N. C., 1954, Eduardo F. Catalano, Architect.

Volume 5 Number 1

STUDENT PUBLICATIONS OF THE SCHOOL OF DESIGN

North Carolina State College

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TWO WARPED SURFACES

Eduardo F. Catalano

With the publication of this material we endeavour to present an approach to the design of space-enclosures, showing a mental process which starts from the basic idea of the indivisible unity "structure-space," through its development from a geometrical, structural, energy-dynamic standpoint, as well as from studies on space organization to create an atmosphere for living.

We also endeavour to interest the designer in searching for other systems than those known as "lineal structures". For those other systems have been used for several thousand years until timber and steel construction directed the design of structures into the limited techniques of joining lineal members with individual structural behavior.

In our attempt to create space enclosures, it is our task as architects to increase our search for methods and techniques to develop three dimensional structures based upon the theory of the membrane, originated in Germany at the beginning of the present century. A mental attitude, the result of the inertia produced by the lack of imagination and courage from designers, as well as pressures from hidden economic interests, has been delaying those developments for more than thirty years.

We present in the following pages a combined study of two double curved ruled surfaces—the hyperbolic paraboloid and the hyperboloid of one sheet. This material is a selection of the study prepared by the fifth year class during two problems. They developed slowly but continuously as information was gathered, collaboration was obtained, and finally built into reality.

The studies began with an introduction to geometry and double curved surfaces. An attempt was made to relate them to surfaces of double curvature from Plateau's experiment on surfaces of low superficial tension. Equations for geometrical breakdown and formulas for design calculation were also presented for practical porposes. We have also included some studies made on the behavior of both surfaces to the action of uniform wind velocity through wind tunnel tests, as well as different ways of organizing the prototype units in space, under the action of light.

We acknowledge the contributions of the engineer, Atilio Gallo, of Buenos Aires, Argentina; Professors Duncan R. Stuart, Robert M. Pinkerton, C. F. Strobel of North Carolina State College; and the Students of the School of Design who are the authors of this article.

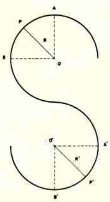


Figure 1

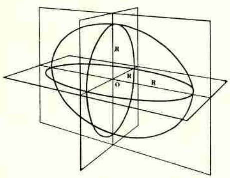


Figure 2

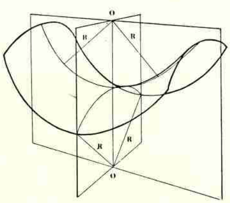


Figure 3

ON GEOMETRY

Since the discovery of texts from the Hammurabi period that date from 2800 B.C. to the present time in which the concept of time is considered, geometry has developed into different comprehensive systems, each one complementary to the rest. They began with Oriental Constructive Geometry which was needed to develop a utilitarian craftsmanship to the contemporary system which expresses a cosmological pattern: a geometry of structures, as the result of active forces impinging on matter, and its reaction following the law of minima and least resistance.

The early geometrical doctrine developed by Euclid in 300 B.C. was the only geometrical language until the seventeenth century when analytical geometry was created, to be followed by the non-Euclidian geometries of Riemann and Lobatschevsky.

Riemann and Lobatschevsky's geometries are instruments for the interpretation of the universe. Both are used only for astronomical dimensions. It has been proven that as long as those dimensions are not greater than several million miles the Euclidian geometry is still valid.

Hyperbolic Paraboloids and Hyperboloids of one sheet, as ruled surfaces of *negative curvature*, belong to the geometry of Lobatschevsky. For our purposes their geometrical analysis falls within Euclidian geometry, analytical geometry, and, in some cases, descriptive geometry which give enough accuracy for preliminary studies.

ON SURFACES

Generally speaking, surfaces can be of different curvatures: null curvature, positive curvature and negative curvature. Their respective geometries are: Euclidian, Riemannian, and Hyperbolic or Lobatschevskian.

The curvature of a surface is determined by its intersections with two normal planes which establish the positive or negative directions of the radius of curvature.

In two dimensional curves the curvature at one point is measured by the inverse value of the radius of curvature $\frac{1}{r}$, which can be positive or negative according to its direction (Figure 1).

In three dimensional curves the surface is positive or negative if the product of $\frac{1}{r} \times \frac{1}{r}$ is positive or negative respectively.

This means that the surface is positive when the radii are situated on the same side of the surface, and negative if they are situated on different sides of the surface. (Figures 2 and 3).

ON RULED SURFACES

A ruled surface is formed by a single infinite system of straight lines.

When two consecutive straight line elements are intersect, the surface formed is plane or single curved, and can be developed.

When the same elements do not intersect, the surface formed is warped and cannot be developed.

Warped surfaces are double curved, but contain straight line elements.

True double curved surfaces have no straight line elements and are curved in every direction: sphere, elipsoid, torus, etc.

The addition of the angles of any triangle on these surfaces is less than 180°.

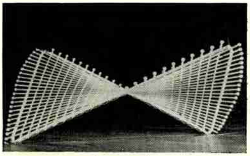
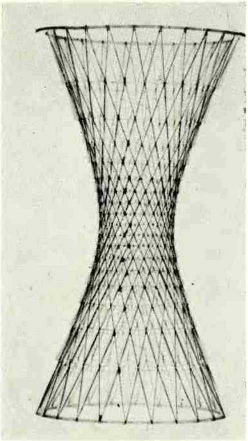


Figure 4



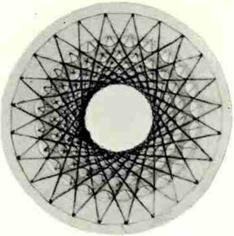


Figure 6

HYPERBOLIC PARABOLOIDS

A hyperbolic paraboloid is a surface generated by a straight line that slides along two straight directrices not in the same plane, and remains parallel to a plane director. (Figure 4).

Vertical planes that intersect a Hyperbolic Paraboloid surface determine parabolas.

Horizontal planes that intersect a Hyperbolic Paraboloid surface determine hyperbolas. (Figure 5).

HYPERBOLOID OF ONE SHEET

A Hyperboloid of one sheet is a warped surface generated by a straight line that revolves around an axis that it does not intersect, maintaining a fixed relation to the axis (Figure 6).

It is also a surface generated by the rotation of a Hyperbola around one axis that it does not intersect.

It is a quadric surface. A quadric surface is such that any plane section of it is conic. It is also called Conicoid.

Facets limited by two intersecting pairs of generatrices are not truly hyperbolic paraboloids, since the generatrices from each set do not displace themselves parallel to plane directors.

In Hyperbolic Paraboloids and Hyperboloids of one sheet, each point of the surface is on more than one straight line.

MINIMUM SURFACES

To become acquainted with different methods of generating surfaces of double curvature, we have carried on some studies on the geometry of similar or quasi-similar surfaces obtained by different causes or phenomena.

The surfaces selected were those generated by a close contour of straight lines and curved lines acting as directrices, to which straight line generatrices and Plateau's soap film methods were applied. We have explained previously the geometrical properties of the surfaces of Hyperbolic Paraboloids and Hyperboloids of one sheet generated by straight line generatrices. We will now explain how the surfaces obtained by using Plateau's methods are generated.

Plateau's experiment consist of the study of surfaces obtained by dipping a closed contour made of thin wire into a soap solution of low surface tension. When the wire is withdrawn from the liquid, a surface of minimum area is formed bounded by such a contour. The minimum surface obtained is due to the action of surface tension created by the effect of forces of attraction existing between the molecules of the liquid.

These molecules are attracted into the interior of the surface by those a little more deeply situated till the process cannot continue and the surface becomes a minimum one (Figure 7). "The molecules of the upper layer are only attracted on one side and are therefore free to exert their energy toward the mass, and their tendency is to bring this into the smallest compass, namely that of the sphere"—Plateau. It is supposed that those surfaces have a stable equillibrium as the gravity force is negligible due to their minimum potential energy[1] from surface tension and also due to their being minimum ones.

---

[1] Potential energy: Energy which a body possesses by virtue of its position. eg. a coil spring, or a vehicle at the top of a hill possesses potential energy. Measured by the amount of work the body performs in passing from that position to a standard position in which the potenital energy is considered to be zero.

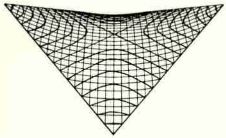


Figure 5

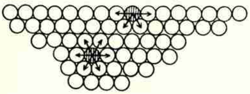


Figure 7

Figure 8

Joseph Henry, secretary of the Smithsonian Institute, in the preface to Plateau's article says: "It should, however, be observed that the force in operation in the phenomena of the heavenly bodies and that in the experiments on soap film are very different, and can only give rise to *accidental similarities*, and not to identical results." Gravity, which is operative in the first case, is the most feeble of all known attractions while its sphere of action is indifinitely great. On the other hand, molecular attraction, which is operative in the second case, is exceedingly energetic while its sphere of action extends only to the nearest contiguous particles, and becomes imperceptible at sensible distances."

Following that method two surfaces of negative curvature were obtained: a double curved catenoid and a catenoid of revolution.[2]

The double curved catenoid (Figure 8) represents a surface almost equal to the hyperbolic paraboloid since catenaries and parabolas, which are respectively the curves obtained by the intersection of vertical planes with either surfaces, are very similar curves.

On the other hand no geometric relationship can be established between the catenoid of revolution (Figure 9) and the hyperboloid of one sheet.

Although sections from horizontal planes in both surfaces produce circles, sections from vertical planes containing the axis produce catenary curves in the catenoid of revolution and hyperbolas in the hyperboloid of one sheet.

Soap film experiments have also been used to sustain astronomical hypotheses such as the formation of Saturn's ring and La Place's hypothesis on the genesis of the solar system.[4]

We present below a description of the several methods applied in studying the soap films formed by sets of straight line directrices and ring directrices, as an observation of the double curved surfaces developed by them.

Figure 8 shows a saddle shaped membrane bounded by a wire frame. Very little geometrical difference is presented between the soap film (double curved catenoid) and ruled surface (Hyperbolic paraboloid).

Figure 9 (top left) shows the surface generated by applying to the two ring directrices a soap solution. Two surfaces are formed, joined by a membrance parallel to the rings. Each double curved surface formed between the rings and the membrane is a catenoid of revolution. By perforating the vertical membrane and gradually detaching the rings, new catenoids are formed, as shown in the other photographs of Figure 9.

Plateau has found that when the separation of the rings is greater than ⅔ of their diameter, the surface loses its stability. The photograph at the bottom right of Figure 9 shows the instant when the catenoid explodes into two concave surfaces which gradually become flat, bounded by the rings. If, before reaching their limit of stability, the rings approach each other, a planar vertical membrane is formed again, dividing the catenoid surface in two, as indicated above.

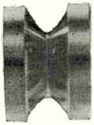
Another method is proposed by Plateau, in which the rims of two glass funnels are used. The equilibrium is formed when the angle between the film and the conical surface of the large funnel is 90°. When the film is extended beyond the limitary height of the catenoid, the surface so originated does not present the least potential energy. (Bottom left Figure 9).

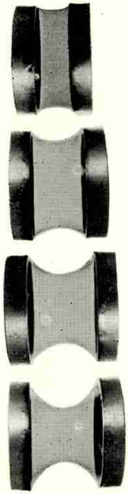
Plateau states in the second part of the same article (annual report of the Smithsonian Institute—1872—page 353): "All catenaries are, we know, alike; surfaces generated by catenaries of different dimension similarly placed in relation to the axis of revolution will be cateniods, of similar figures. The complete catenoid is not susceptible to variation of form but constitutes a unique figure, like the sphere and the cylinder. Hence the catenoids which, theoretically, (for simplicity the two rings or edges are considered without thickness), rest on the same rings, when the separation of these is below the limit, do not differe from one another except by their dimensions absolutely homologous."

[2] Catenoid of revolution is a surface originated by the rotation of a catenary curve.[3]

[3] Catenary: curve formed by a chain or a string hanging from two fixed points. Its formula is very close to that of the parabola.

[4] Annual report of the Smithsonian Institute—1864, page 207: "The Figures of Equilibrium of a Liquid Mass" by J. Plateau.





GEOMETRY FROM HYPERBOLIC PARABOLOIDS

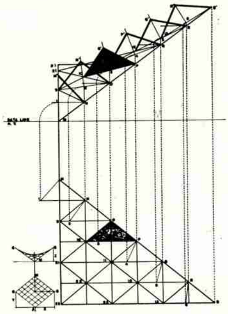
Hyperbolic Paraboloids, as all the ruled surfaces of compound curvature, are not developable. Figure 10 shows how we have approximated their developed surface by triangulation. In this particular example, true lengths of members and true angles were obtained by descriptive geometry. A more accurate way to obtain the same result is by using analytical geometry methods. For this purpose, a formula is given to position any point on the surface, related to a horizontal plane of reference $Z = 0$. The other values—distances between points, angles formed by two meeting straight generatrices, etc.—can be easily found by trigonometry.

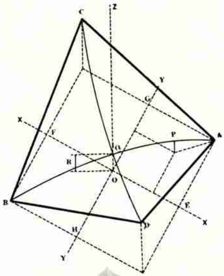
HEIGHT FOR ANY ON THE SURFACE (Figure 11)

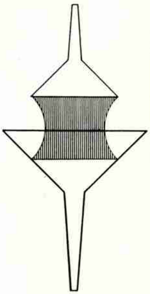
Equation of a theoretical surface for a rectangular hyperbolic paraboloid:

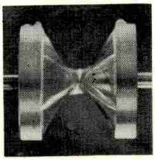
$$Z = A - B \times X \times Y$$

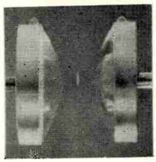
$Z = 0$, height of plane of reference
$A =$ height of central point (fleche)
$X =$ half of length of side X axis
$Y =$ half of length of side Y axis

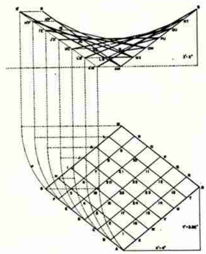


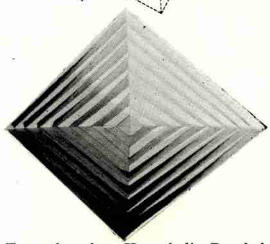












Even though a Hyperbolic Paraboloid is not a developable surface, there is a way of obtaining it from a flat single sheet of paper, as shown in Figure 12, by twisting each strip of paper during the folding operation.

Above Figure 9
Center Figure 10

Left Above 11
Left Below 12

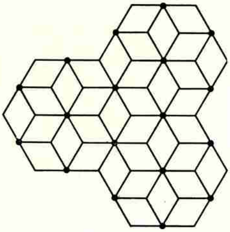


Figure 13

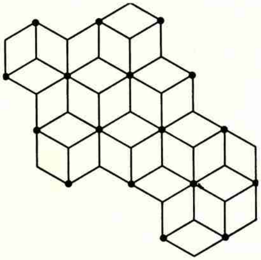


Figure 14

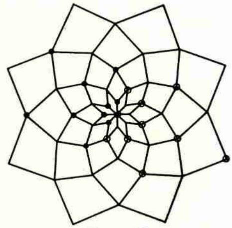


Figure 15

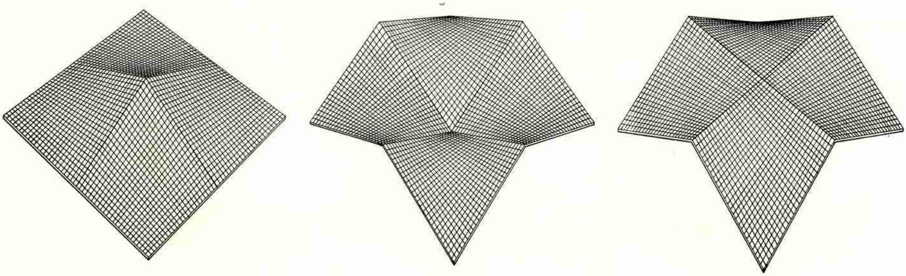
COMBINATION OF UNITS

A variety of geometrical patterns can be organized by grouping the units in different ways which take advantage of their variables: lengths of generatrices and directrices, true angles, and heights of vertices, as shown in the following figures, from 13 to 25. In some patterns there is a basic symmetrical unit, and in others different units combined.

Most of these examples are simple combinations of units. The fluidity developed by the warped surfaces with their up and down curvatures requires, in almost all studies, a visual appreciation through the construction of models. As previously stated, light plays a fundamental role in the definition or reading of warped surfaces.

We leave to the reader the further study of the limitless combinations of units, especially those which because of their relationship present a series of horizontal generatrices and directrices which allow a simple subdivision of the total space, avoiding arbitrary intersections between the surfaces and vertical dividing planes.

Figure 16



ORGANIZATION OF UNITS

In their organization, the units can have, from the *Geometrical* point of view, any position in space. But *structurally* speaking, their positions are limited by the type of stresses developed. As we will discuss later, the Hyperbolic Paraboloid shell stands only stresses of tension and compression when the Z axis is vertical and with uniformly distributed loads. Tilted shells (Z axis inclined) will be subject to bending.

Combinations of several units present several structural advantages. While the calculation of the membrane and the peripheral edges remain the same as in individual Hyperbolic Paraboloids, the common edges, although under twice the compressive stresses of the individual one, sometimes present a better section to resist buckling because of the angle formed by the joining surfaces which creates a virtual depth.

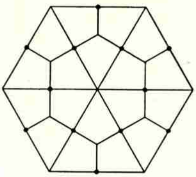
In some cases forces acting at the low vertices of a unit are equilibrated by equal forces from the opposite unit.

Figure 13 shows a combination of rhomboidal hyperbolic Paraboloids. Each set of three forms a hexagonal unit with the high vertices at its center.

Figure 14 shows the same system organized in a freer order.

Figure 15 shows a spiraling organization of units of different dimensions. In this figure, two positions of the low vertices (supports) are indicated: Black points and circles show that the same plan can originate different spaces by changing the position at the supports.

Figure 16 shows three combinations of four units.



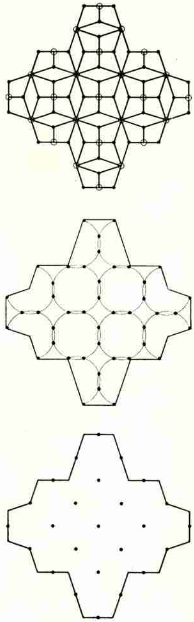
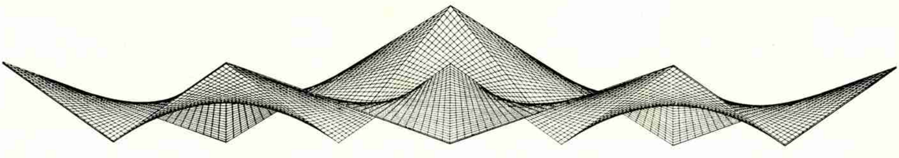


Figure 18

Figure 17



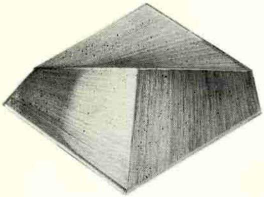


Figure 19

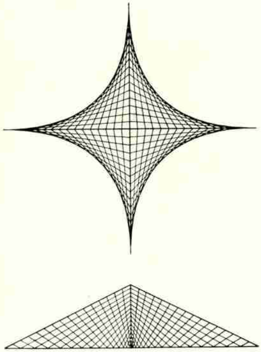


Figure 21

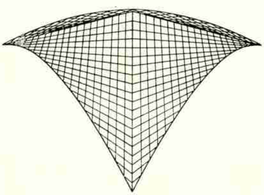


Figure 23

Figure 17 shows a combination of two different units. By positioning the supports at different vertices, two different sequences of spaces can be formed as shown in the drawing. In the top figure, black points at the vertices of the small units result in the space organization as indicated in the second figure, and white circles situated at the other set of vertices result in the space organization as indicated in the third figure.

Figure 18 indicates a plan of eighteen units with its elevation shown below. The same plan can create different spaces by changing the height of the vertices.

Figures 19 and 20 show three rhomboidal hyperbolic paraboloids combined as a hexagonal unit. As the perfectly symmetrical structure so obtained has three supports and its central high vertex is rigidly connected to them by three edge-beams, its structural stability is very favorable against wind action.

Figure 21 shows four half-tilted hyperbolic paraboloids. Although their Z axes are not vertical, the fact that each horizontal parabolic edge rests on the ground makes this combination a structurally efficient one.

Figure 22 shows a space defined by different types of hyperbolic paraboloids.

Figure 23, 24, 25 show three other combinations. The few examples shown here give a clear idea of the endless variety of spaces that can be created by the different grouping of units, by the positioning of the supports, or the height of the high vertices.

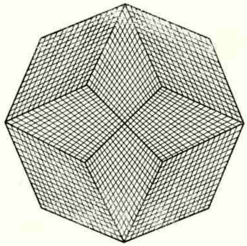


Figure 24

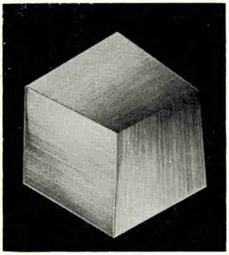


Figure 20

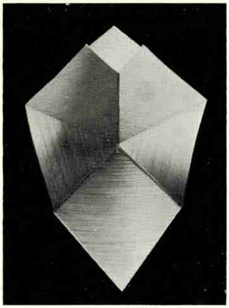


Figure 22

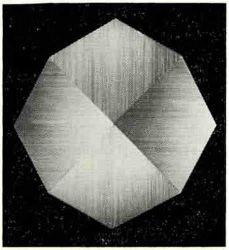


Figure 25

STRUCTURE

In this chapter we present studies made by the French Engineer, Aimond[5], on the behavior of the surface of rectangular Hyperbolic Paraboloids for uniformly distributed loads.

Before the availability of these studies, a loading test was carried out at the laboratories of North Carolina State College, which gave only an idea of the direction of the internal stresses. We omit some of our findings, since Pilarski presents in his book *Voiles Minces*—1935, a very comprehensive explanation of the internal stresses, and formulas for the calculation of hyperbolic paraboloids, based on Aimond's studies.

A)—Extracts from Pilarski's book: *Voiles Minces*

The hyperbolic paraboloid is a surface with equal structural behavior for uniform distributed loads parallel to its Z axis.

On every point of the membrane the internal stresses are R_1 = tension stress
R_2 = compression stress

The values of R_1 and R_2 are constant in every point of the surface.

On each point of the edge of a hyperbolic paraboloid the internal stresses R_1 of tension and R_2 of compression are composed as tangential stresses which are equilibrated by the reaction of the edge.

Every section parallel to AC is a parabola subject to simple compression.

Every section parallel to DB is a parabola subject to tension.

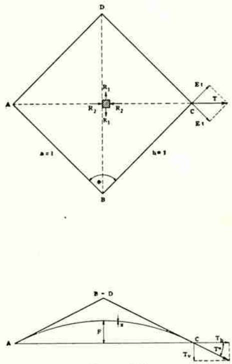


Figure 26

Formula for stresses at every point of the surface (per square foot). (Fig. 26).

$$t = \pm \frac{wab}{4 \times 2 \times F}$$

w = Live and dead load per unit area
$a = b = 1$ = Length of edges
F = Height of middle point of surface.

For rectangular hyperbolic paraboloid $\Theta = 90°$

$$t = R_1 = R_2 = \frac{wab}{4 \times 2 \times F}$$

The total tangential stress on the edges to be equilibrated is E(t), where E = length in feet of edges a and b.

In a hyperbolic paraboloid the thrust to the void on the straight edges is null.

The fatigue of the material of the membrane under the action of its own weight is independent of the thickness of the membrane, but the live load (snow, wind) and the technique and material adopted.

MEMBRANE CALCULATION

On every point of the membrane the internal stresses have a constant value for load uniformly distributed:

We have seen that:

$$t = \frac{wab}{4 \times 2 \times F} \qquad S = \frac{t}{m \times C} = 12'' X \qquad \times = \frac{t}{12'' \, mC}$$

S = Section of membrane per lineal foot (12x inches).
t = Stress at every point
m = Resistance of material
C = Coefficient of safety
x = Thickness of membrane

B)—1. The stress in a hyperbolic paraboloid are only tension and compression. Their values are the same at exery point of the surface.

[5] Aimond, Office of Bases, Minister of Air, France.

2. No bending exists if the surface presents its **z** axis vertically.
3. Stresses of tension and compression are resolved as tangential stresses along the edges.
4. The nature of the tangential stresses (compression or tension) depends on the position of the supports. Compressed edges result from supports at the low vertices. Tensile edges result from supports at the high vertices.
5. Compressed edges behave as columns with axially increasing load from high vertex (zero) to low vertex (maximum).
6. The membrane calculation is of minor importance—the system adopted as support is more fundamental.
7. The greater the surface curvature, the lighter the structure becomes. Remembering that:

   $t = \dfrac{wab}{4 \times 2 \times F}$ t decreases when F increases.

8. Symmetrical hyperbolic paraboloids present better static conditions. This concept applies to all structures. As Mach says, "The forms of equilibrium are often symmetrical and regular. Every deformation in a symmetrical system is complimented by an equal and opposite deformation that tends to restore the equilibrium."

Calculation: Example of a square hyperbolic paraboloid supported at the low vertexes A and C. Figure 26.

$E = a = b = 100' - 0''$

$F = 10' - 0''$

w = dead load and live load 40 p.s.f.

$$t = \frac{wab}{4 \times 2 \times f} = \frac{40 \times 100 \times 100}{4 \times 2 \times 10} = 5{,}000 \text{ p.f.}$$ (Tangential stress developed per lineal foot)

$Et = 100 \times 5000$ p/f $= 500{,}000$ p (Compressive stress to be absorbed by each edge)

$T = Et\sqrt{2} = 500{,}000\ (1.4142) = 707{,}400$ p (Stress at A and C tangential to the membrane, to be absorbed by the foundation perpendicular to the direction of the stress)

For foundations not perpendicular to T:

$T_h = T \cos \propto$ (tension)

$T_v = T \sin \propto$ (compression)

EDGE CALCULATION

S = Area of edge section

I = Moment of inertia

P_c = Critical load

E = Modulus of elasticity

Et = Tangential load

C = Allowable stress of selected material

l^2 = Length of edge

$$S = \frac{Et}{C}$$

$$P_c = \frac{NEI}{l^2}$$

(Values of N vary according to shape of edge and distribution of loads. Values are obtained from table on page 139 of *Theory of Elastic Stability*, by S. Timoshenko.)

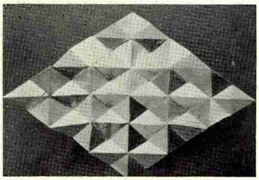


Figure 27

STRUCTURES AND RIGID FORMS

A form has a structural quality when it's matter is organized following certain laws—law of minima, law of least resistance to the impinging forces (gravity, dead and live loads, etc.). Many forms present enough rigidity to stand the mentioned forces, but the organization of their matter does not follow such laws. We differentiate them from structures by calling them rigid forms. The following example illustrates this principle.

Figure 27 shows a model of a hyperbolic paraboloid built with two faceted sheets of metal. The rigidity is acquired by the organization of the facets and the continuous staggered depth obtained by the overlapping of both sheets.

The approach given to the solution is suitable for planar structures where bending moments occur but not for surfaces of double curvature. The organization of matter does not follow the nature and pattern of the stresses. Thus a rigid form has been obtained but not a structure.

PRECAST CONCRETE SHELL AND METAL LATTICE WORK

The following three figures show the result of studies made to construct with different materials and techniques the same structure.

Figure 28 shows a model built with precast units each one with its own set of dimensions, angles and pitches. Each precast unit has a lightening void which, although not structurally necessary since the stresses are equal at every point of the surface, has been designed as a study of the action of light on textured surfaces and as an attempt to disperse sound waves in a more different way.

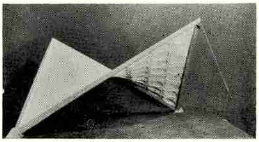


Figure 28

Figure 29 shows a loading test carried on a lattice work hyperbolic paraboloid without stiffening edge. Figure 30 shows the same model after the edge has been added, which gives considerable undeformability to the lattice work.

TESTING MODEL

Figure 31 presents the loading test of a balsa wood laminated model. It was built with 9 laminations each 1/16″ thick. The model presented such rigidity that a uniform load of 100 times its own weight was unable to crack the coat of paint applied on both surfaces to determine positioning of the strain gages used for the final test and readings.

AIRDYNAMIC TESTS

The irregularity of the surface created by a group of hyperbolic paraboloids, with their up and down curvature, their high and low vertices and the variety of surface behavior under different wind directions, made wind tunnel tests a study of interest. Several types of tests have been carried out: wind tunnel tests to determine the surface behavior to different wind directions and velocities for structural purposes; and wind tunnel tests for study of problems related to natural ventilation of the space enclosed.

Figure 29

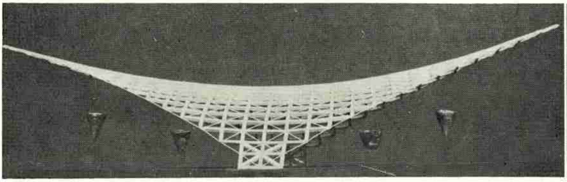
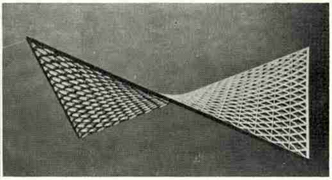


Figure 30



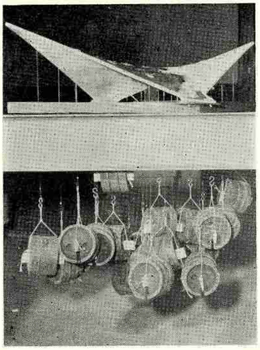


Figure 31

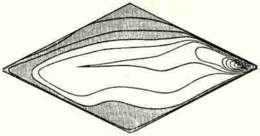


Figure 32

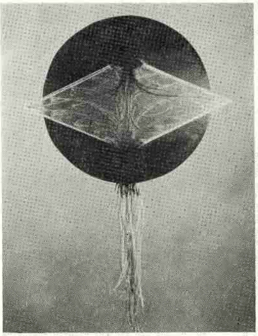


Figure 33

WIND TUNNEL TEST.—STRUCTURE[6]

Stresses due to the effects of wind are a major design consideration in any large structural system. In structures of single or double curved surfaces, it is possible for wind action to affect a complete reversal of stresses within the form. Acting as air-foils, these surfaces tend to distribute wind loads in a manner directly opposite to the distribution of static loads. In many cases, structural elements which ordinarily would be designed for a light tensile load due to dead and live load reactions, might also have to be considered as taking a heavy compressive force under wind action. This is readily understood if we investigate how air flow over the surface of an air-foil affects the stresses on it. As air flows over the surface, a decrease in static pressure takes place, becoming smaller as the velocity of the wind increases. This decrease in pressure may be regarded as a lifting force or suction on the surface of the wing, acting upward and normal to the surface. In the same wind velocity, a slightly curved surface will exhibit more lift than a flat surface, since the air tends to flow faster over the curved surface, usually approximately doubling its original velocity by the time it reaches the high point of the curve; then decreasing to the original velocity. For the sake of simplicity, and because it bears a closer relationship to problems of structural design, only the top surface of the air-foil has been considered. Generally speaking then, an increase in the air velocity over a surface results in a decrease in static pressure, causing a lifting force.

In investigating the aerodynamic properties of the hyperbolic paraboloid, several conditions of wind effect were studied. These effects were both negative and positive (pull and push, respectively) pressure distribution and critical points, total lift on the surface, over-turning moment tendency, center of pressure location, and the study of air flow through and around the form. All forces on the shape were considered by determining the force distribution on a scale model of the hyperbolic paraboloid unit (see Figure 33) tested in a closed-circuit, single-return wind tunnel. Forces and pressure coefficients on the model were analyzed by the pressure distribution method using a multiple tube manometer. Conditions on the model tested were most critical when the air flow was directed at approximately 45° to the major axis of the model. This position, high negative pressures were recorded on the top surface at the point into the wind (See Figure 32). A positive pressure on the bottom surface at the same point was also recorded tending to increase the total lifting force in this area. When the major axis of the model was parallel to the air flow, a large over-turning moment was exhibited, affected by a large concentration of positive pressure on the portion of the bottom surface into the wind. A relatively high negative pressure distribution on the top surface took place in the same area, supplementing the positive forces. At 90° to the air flow, fairly uniform negative pressure distribution was affected on both surfaces.

Figure 33 shows the model tested for static pressure distribution in the wind tunnel of the Aeronautics Department of North Carolina State College. It was tested in three positions: parallel, perpendicular, and at 45° to the direction of air flow, each position being tested at three speeds (approximately 50 mph, 75 mph, and 100 mph). Twelve tests were run in all, nine on the top surface and three on the bottom surface. The bottom surface was tested in three positions, but only one air speed, since pressure distribution on the bottom surface is relatively unimportant, assuming that the space covered by it will be enclosed.

[6] From wind tunnel report by John Caldwell and Joseph Constanza.

WIND TUNNEL.—VENTILATION.

A model of four hyperbolic paraboloids of extraordinary rigidity made out of dowels and silk base paper was tested under three different wind directions.

The photographs in Figures 34 and 35 show the models inside the wind tunnel for wind directions along the two major axes.

The action of the wind in each case was defined by the different angles of turbulence produced by the tufts distributed regularly on the surface.

Figure 36 shows the test in which the wind direction was parallel to the longitudinal axis. The steady tufts indicated by a single line show areas of negative pressure while the revolving ones define areas where the wind plays on the surface without affecting it. (See gray areas)

The tests gave valuable information in finding an efficient location of air exhaust for natural ventilation of the space enclosed by the four units.

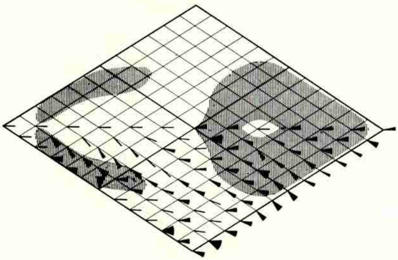


Figure 36

Figure 34

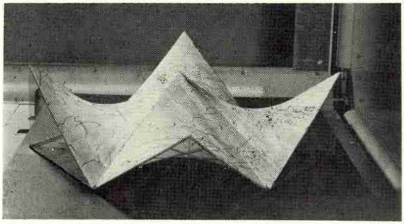
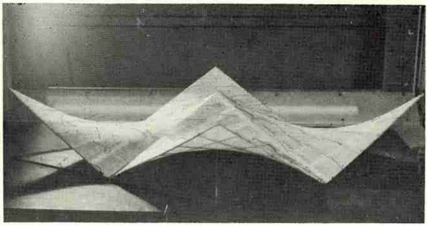


Figure 35



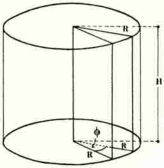


Figure 37

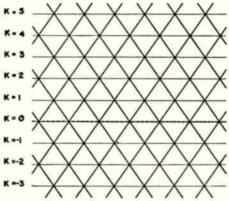


Figure 38

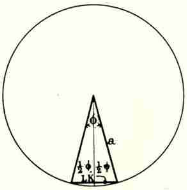


Figure 39

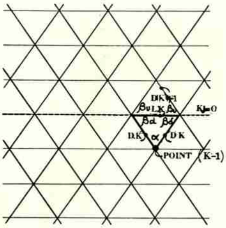


Figure 40

GEOMETRY OF HYPERBOLOIDS OF ONE SHEET

Collaboration with Prof. C. F. Strobel, Department of Mathematics, N. C. State College

Consider that portion of the circular hyperboloid of one sheet between two right sections located symmetrically on each side of the center of the hyperboloid. Let these sections be H units apart; circular sections have a radii of R units. Also, consider f strings fixed at points on the circumference of these sections so that they run parallel to the axis and are equally spaced around the cylinder having the circular section as base. (Fig. 37).

There will be one string at every $\frac{360°}{f}$ around the circle. Call this angle ϕ:

$$\phi = \frac{360°}{f}$$

Now if the bases are turned around the axis by an angle, ϕ, relative to each other, the strings become "skew" rather than parallel and become one set of elements of the hyperboloid. Assuming the strings to be inextensive, the circular bases will draw together and be separated by the distance (b):

$$b = \tfrac{1}{2}\sqrt{H^2 - 4R^2 \sin^2 \frac{\Theta}{2}}$$

THE PROBLEM: Give strings of a second set a twist equal to angle of twist of first set but in opposite direction and then determine formulas for the distances between the intersections along one string with those of the other set, and to find the angles of intersection between members of the two sets.

Strings of the two sets must meet each other at points on the circular bases where they are fixed. Thus Θ must be an integral multiple of ϕ.

$$\Theta = n\phi = n\frac{360°}{f}$$

Note: n an integer.

Could use $n = \frac{p}{2}$

where p is integer.

1. Each element of one set meets those of the other set in

$$2n = \frac{2\Theta f}{360°} + 1 = \text{number of intersections.}$$

2. There are facets arranged vertex from bottom base to top base so that these vertices lie in a plane containing the axis of the hyperboloid and there are n of these, or:

$$n + 1 = \frac{\Theta f}{360°} + 1 = \text{number of vertices}$$

To use distance formula we notice that K is the only variable. K = +I gives distance from midpoint (K = 0) to the next higher intersection, K = +4 gives distance from the 3rd point above midpoint to the 4th point and so forth. (Fig. 38).

ϕ = Central angle subtended by points where strings are attached $\phi = \frac{360^\circ}{f}$ (Fig. 39).

a = Radius of circular section of k level which pases through points of polygon. (Fig. 39).

$$a = \frac{R \sin n\,\phi}{\sin n\,\phi + \sin\,(n + K)\,\phi - \sin K\phi\,\sqrt{4[1 + \cos(n + K)\,\phi}}$$

L_k = Length of chords to right circular section of level K. (Fig. 39).

$$L_k = 2\,R\,\frac{\cos \frac{n\,\phi}{2}}{\cos \frac{K\,\phi}{2}}$$

16 Angles on the under side adjacent to chord, made with elements of either set by chords on any right section are called β_d and are all equal; those on the upper side are called β_u and are all equal. (Fig. 40).

$$\propto = 2\sin^{-1}\frac{L_k}{2\,D_k} \qquad \beta_d = \cos^{-1}\frac{L_k}{2\,D_k}$$

$$\beta = 90^\circ - \frac{\propto}{2} \qquad \beta_u = \cos^{-1}\frac{L_k}{2\,D_k + 1}$$

$$\propto K\text{-}1 = 180^\circ - 2\,\beta_d$$

WIND TUNNEL TEST OF HYPERBOLOID OF ONE SHEET

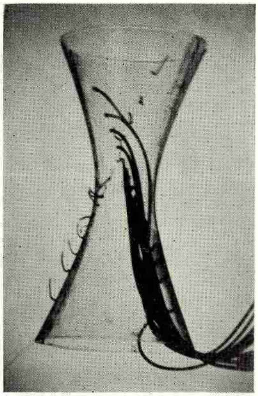
Aiming at similar results to those obtained through the wind tunnel test of hyperbolic paraboloid, a Plexiglass model of a hyperboloid of one sheet was made as shown in Figure 41. The orifices on the surface for connection through plastic tubes to the manometer, as well as the tufts, were located along two of the generatrices of the surface. Readings were obtained over the complete surface by rotating the hyperboloid of one sheet 24 times.

Figure 42 shows the fluttering of the tufts on the outer surface, while Figure 43 shows them on the inner surface. In both drawings the generatrices have been curved in order to present the undeveloped surface of the hyperboloid of one sheet as a flat diagram. Steady tufts are indicated as short straight lines; black sections of a circle show various degrees of fluttering; white circles indicate a complete rotation of the tufts.

As in the case of the hyperbolic paraboloid tests, the steady tufts determine areas of negative pressure, while the other tufts determine areas of different degrees of wind action. The wind has no effect on the surface where the tufts are shown as full rotated.

Figure 44 shows 17 horizontal sections of a hyperboloid of one sheet, each one indicating the results of the readings from the action of the wind on the surface.

Figure 41



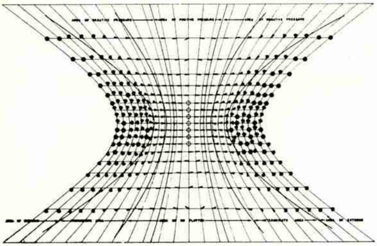


Figure 42

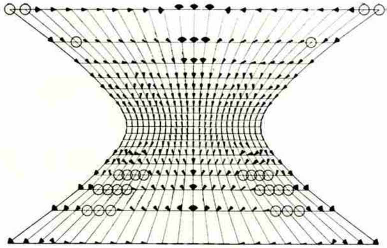


Figure 43

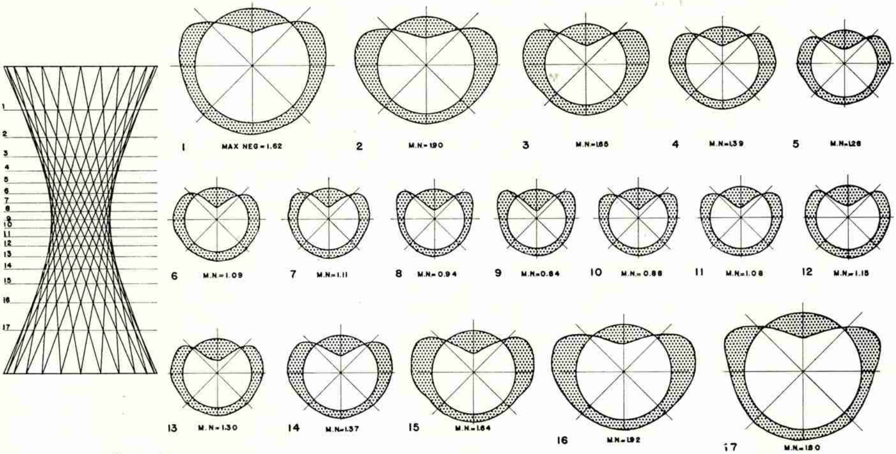


Figure 44

The following students have participated in the research that made this article possible.

Class 1953 S. A. Allred; R. G. Anderson; J. L. Bennett; W. E. Blue, Jr.; J. T. Caldwell; T. J. Condit; J. Costanza, Jr.; C. H. Dorsett, Jr.; J. H. Hammond; S. C. Hodges, Jr.; B. Leon; L. H. Mallard; J. P. Milam; R. Miller; R. T. Mitchell; S. Pardue, Jr.; G. M. Slack; C. M. Taylor; F. M. Taylor, Jr.; D. B. Winecoff.

Class 1954 R. C. Bates; B. J. Blech; W. R. Campbell, Jr.; F. W. Coile, Jr.,; W. L. Crouse; E. G. Egan; C. E. Gerrald; K. Goldfarb; J. M. Jenkins; J. S. Kina; R. L. Knowles; M. K. Nakayama; T. J. Peters; F. Rolnik; P. S. Shimamoto; F. V. H. Smith, Jr; M. J. Sperber; H. J. Spies; G. T. Willis.

Drawings By: R. L. Jackson; R. P. Leaman; G. A. Buff; R. B. Tucker.

Felix Candela

STRUCTURAL DIGRESSIONS AROUND STYLE IN ARCHITECTURE

Felix Candela, Mexican Architect, engineer and builder internationally known for his concrete thin shell designs.

The professions of Architect and Engineer, once united under the title "Master Builder," have widened to such a dangerous extent that today few dare to tread the no man's land between them. Yet, on those numbered occasions when someone has had the courage and talent to take his stand there—such as Maillart and Nervi from one field and Nowicki and at times Wright from another—the results have been so extraordinary as to force us to consider whether it is not there, finally, that lies the hidden solution to the fundamental architectural problem of our age. This problem is, in my oppinion, the search for a style or common language able to offer us something more than the aridness of mere routine.

A style is established when abstract forms begin to acquire a symbolic significance derived from time and custom. It is evident that to the layman all architectural forms appear abstract in content; but the architect himself must distinguish between those abstract by effective definition, such as the moulding or decoration in general, and those expressive of the structure which must be his essential preoccupation when he comes to consider the material means of enclosing a given space. When he achieves their just equilibrium, when the structural, or necessary, is in balance with the decorative or superficial, he creates a true architecture.

Revolutions, after attaining their primary objective of overturning a previous "status quo," must establish a basic code backed by one political and philosophic programme. In the case of Architecture, such a code is called style. The architectural revolution of the twentieth century has executed a near total destruction of the former practices whose ineptness demanded the outbreak; but we have yet to witness the constructive phase. Perhaps this phase has been retarded by the absolute freedom we have enjoyed in the attempt to evolve a new style, and perhaps by the continuation, albeit in a different manner, of the long established habits we condemn. Freedom itself can seldom produce positive results, since the middle-man may adjust himself only within fixed limits which, while curtailing his liberty of movements, give him the confidence and security essential for his competent function. The genius alone is capable of living happily in such a rarefied atmosphere, and to him alone is given the power to create, under such conditions, works that automatically constitute law; works, that is to say, that will form the grammar and vocabulary of a new style. The inherent danger is that he may dedicate himself to expressions which, although bearing the indisputable stamp of his talent and vitality, may not be based on authentic architectural values. Thus a senseless formalism may be created that will later require a disproportionate effort to abolish. This process has been continued casually for several decades, seemingly unnoticed.

It might be convenient here to compare very briefly what occurred in the formation of the historical orders. For the major part they comprised little but modification and adaptation of preceding styles, but two, the Gothic and the Greek, may be considered as original and scrutinized for the purpose. It is ironic that Gothic Architecture, considered through centuries an incult and barbaric art, should constitute the main manifestation of lucidity in the history of Western building. Here we find a true architecture characterized by the subtle blending between the building and its sculpture, between the pure structure and those decorating or adorning elements used as a further accent of the structural solution. However, the various attempts to revive Gothic in the last hundred years have always halted at the superficial and external aspect of the problem, without trying to penetrate the profound significance of its philosophy, its intimate combination of structure and expression.

In contrast Greek Architecture, or classical composition "par excellence," is as little in the category of architecture as it is sculpture. The Greek enjoyed a gentle climate and his life, social and commercial, was enacted in the open street. As a result his approach to architecture was one of outdoors, where his buildings were most prone to appraisal. He had little interest in interior space, hence less in the constructional methods used to create it. His buildings were not formed in the embodiment of a structural logic, but in the reflection of ritual customs and symolism, executed in the most permanent material he could find. The fact that the post and lintel, copied from the traditional forms of primitive wood structures, is one of the most absurd ways to employ stone was apparently of no concern to him. It is a paradox that the Greeks, so advanced in all other facets of culture and science, should have been, in this respect, so blind. Greek temples, stylised reproductions of early wooden temples, with their monstrous pillars supporting nothing, may be admired for their impeccable carving and elegant proportions; but they are rationally explicable only as pure sculpture. Several centuries of a hesitant process of adaptation had to pass before the arch, the most logical system of construction in stone, was again generally accepted.

In recent times the same process of adaptation was applied when steel appeared on the scene, although it was less evident since the basic structural element of steel is the elongated prismatic member, in which the length predominates over the other two dimensions. This is similar to the wooden structural member, and therefore there was little reason why both materials should not be used in similar manner.

Reinforced concrete is a much newer technique. Its extensive use began only in the late nineteenth century, coinciding with the climax of steel construction whose possibilities were by then fully developed. The initial progress of its adaption has seen its employment in forms directly copied from the structures of steel and wood, even to the imitation of their systems of analysis.

Now a circumstance did exist to justify the Greek in his treatment of stone, in the symbolic importance that the reticulated wood frame had acquired in the eyes of his people. A similar justification cannot be extended to excuse the foolishness of the current approach to reinforced concrete design. The skeleton frame in reinforced concrete is a structural manifestation almost as inconsistent as the stone lintel, being in addition a dull and routine copy of the structural forms of wood and steel. Concrete is not made to work in beams of massive rectangular section. Its highly disfavorable ratio of resistance to weight quickly limits the span this kind of structure may achieve, as most of the beam is acting only as a dead weight with no structural or resistant function. By mistaking the mere possibilities of the material for its real properties, the foundations of the formalism predominating contemporary composition have been laid, on the basis of the purely literary pretensions of functionalism of earlier times.

It is noteworthy that the aphorism "form follows function" should have attained such an awesome public stature. In reality, of course, the end behaviour or function of a material will depend entirely on the form given to it. If modern architecture is but the varied play of barren rectilinear systems, it is perhaps because the closest practical interpretation of Sullivan's dictum is resolved through form following "the fashion." It would be a tragedy if the production of cubic masses, arranged into rectangular planes of glass and accentuated by murals and plants, should win a recognition synonymous with architecture in the minds of the new generation; or if the monotony achieved by endless repetition should be regarded as a valid expression of unity and style.

It is now time at last to begin the constructive phase of our architectural revolution. The "International Style," as epitomised in the recent erection of cubist abstractions, has nothing further developable to offer us. The classical and destructive phase should be considered complete.

For lack of a strong spiritual motivation—whose default is also responsible for the confusion prevalent in so many other aspects of modern society—on which to base the new style, it would seem logical to confine our reasoning to more materialistic considerations. These correspond, by very definition, to the structural element in architecture.

At present, architects do not seem attracted to the problems of structure. Excusing themselves on the grounds of their preoccupation with the superficial design and arrangement of modern mannerisms, they expect conveniently that some one will oblige to solve the problem for them. But engineers (the indicated party) have a like disinterest in the problem. Their chief concern is to justify the boring and insignificant forms presented to them by means of a mechanical process that has nothing to do with design; that is, to calculate these forms.

Calculation is a mathematical process in whose application it is necessary to simplify —abritrarily in most cases—the properties of constructional materials. It constitutes one of many means—and not the most trustworthy—of ascertaining in rough approximation that the form and dimensions adopted for the structure will be generally acceptable to the common criterion, and consequently that, so calculated, the structure will present a certain probability of stability and permanence. Any pretense of obtaining "exact" estimations is an absolute illusion. On the contrary, such estimations can ostensibly justify the greatest structural blunders; but they are incapable of giving us the proportions of a structure, since both its form and dimensions must have been previously determined at its conception by design. Design, and structural design of course, is an intellectural process of synthetic nature, in which is to be found imagination, intuition and experience, and which demands a certain freedom in the creative agent. It adheres, in short, to the same laws as those of artistic creation. Thus it presents to some minds the inconvenience that such laws cannot be included in any chapter of the Building Regulations.

Major sciences are fully aware of the need for this synthetic process, which is the basis of all development. Unfortunately, it would appear that the minor sciences, and principally the science of structure, are still lost in a jungle of analysis, whence they will escape only through periodical revisions of their basic hypotheses. We are now working under the same assumptions developed more than 100 years ago by several synthetically-minded French thinkers. The fruitfulness of such hypotheses in their time does not guarantee their everlasting immutability. Yet, every paper for an engineering Society must be restrained today to a dry exposition of what are considered facts; as if the Secretary of any technical association were in formal possession of eternal truth. Personal opinions are absolutely forbidden.

Moreover, it is curious to note that the least scientific part of engineering is that to which calculations refer; for although mathematics intervene in them, and often advanced mathematics at that, the process is essentially reduced to the mere application of a Code. Hypotheses of doubtful value are indiscriminately accepted, since without these the interpretation in mathematical terms of the building materials would be quite impossible. But the essence of scientific investigation is the constant doubt, the diffidence of the results achieved, and the eternal pursuit of the unattainable reality. It follows that only the experimental, or empiric, part of Engineering has the right to consider itself Scientific.

The architect of today seems willing to experiment with any of the elements that combine to make a building—except that which should be his first and most vital concern. Towards the structure he maintains the same classical outlook that characterized the Greeks. When he forges this missing link in the chain of philosophy governing his present designs, and not until then, he may justify the hard struggle for an authentic Architecture fought in the earlier part of this century, and he will acquire the tools to fulfil the promise that the opportunities of our age demand from him.

Meanwhile we continue to endure the innocuous constructions in whose indulgence, as an architect, he may be endangering the future of his very existence. For such is the aspect of contemporary buildings that to the common eye they might as well have been produced through a machine, in which some fault of mechanism results in the production of near identical solutions for highly diversified problems.

It is easy to discern in current architecture a preference to treat the structure as the predominent feature of the composition. The supporting framework is bared, and breaks through the surface of the facade. It is forgotten that the mere disclosure of the structure is not a guarantee of its intrinsic beauty. Nevertheless, this tendency may prove to be the mark of the new period of integration in architecture, that is when the structure once comes to receive the just and sensitive treatment it deserves.

In every historical period, architectural composition has taken its stand with stronger insistence on one or another of the three fundamental values—function, structure and form—whose happy integration produces a true work of architecture. It appears time to end a period in which function has held the reins of importance; its limited capacity for creating forms will soon be exausted, and it has led unfailingly to aridness of expression. We remain with structure as the only logical element able to impart a general sense to architecture; able to grammatize a language easily understood by all; able to produce, finally, expressive forms characteristic of a style whose emotional content would depend on the only stimulus able to generate sensitive reaction in these times of crisis: human reason.

The situation offers to architects a real challenge to enter a field which may have looked before far to complex to be explored. Perhaps this complexity is only apparent. The imminent initiation of another revolution in the so-called science of structure would indicate the propriety of an architect's tread-on the sacred ground. I would say that the immediate task of the building profession is to close this wide gap between architecture and engineering. In such an endeavour, the architect might retrieve his lost title of "Master Builder."

ON THE ORDERLY SUBDIVISION OF SPHERES

Duncan R. Stuart

Duncan R. Stuart, Associate Profesor of Design at the School of Design, North Carolina State College is illustrator of his own article.

For one reason or another men have preoccupied themselves for a considerable period of history with questions concerning the geometrical properties of spheres - or perhaps, more generally, with problems of compound curvature. This preoccupation has gained much subtlety as knowledge of the external world has deepened. The sphere seems to have occupied a unique and at times mysterious position in the processes of geometry, bound as it is to the incommensurable "pi" and to the fact that it does not easily lend itself to that kind of a geometrical extension known as a coordinate system. In these and other ways it has presented an obstinately intractable position in the geometrical heirarchy.

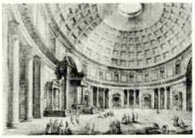
Nevertheless, as experience has increased, men have become aware of the spherical nature of their world - and aware that knowledge of the properties of sphericalness can afford great advantages. In one direction it has been found useful to project ideas concerning the sphericalness of earth into various forms of maps, as aids to coordinate exploration and mastery of environment. In another direction, history shows us abundant evidence of men's use of sphericalness as a structural order. The history of this form as an object of beauty and utility is an exceedingly rich one. We encounter it in seemingly endless variation in nature - and the history of technological man shows it in lengthy and diversified employment.

Under these circumstances it would seem that men's scrutiny of nature would have lead them to a greater variety of methods for modulating spherical configurations than is actually the case. For though the analytical tools for dealing with this problem have been available for a considerable time, imaginative inertia seems to have kept us from going farther than we have. Our preoccupation with the translation of methods for dealing with flat surfaces onto those of spheres has tended to obscure our vision. Because of this geometrical bias we have been inordinately attracted to the latitude-longitude system as a means for ordering sphere - but, admirable as this system may be, for certain kinds of problems it leads to well-nigh hopeless complexities.

Our purpose then will be to show some of the basic properties of spherical order and to indicate some ways that these properties may be used. By doing this we hope to present the designer with a broader spectrum of choices - choices which will enable him to more fully optimize certain geometrical properties, i.e., minimization of number of kinds of faces, vertexes and edges, minimization of difference in area of face, length of edge or difference in angular conditions, maximal standardization of conditions of joining, etc.

Though this short paper cannot pretend to completeness in these matters, it is hoped that it will indicate the broad outlines of a methodology in sufficiently clear fashion that readers with normal first year training in college mathematics can procede on an independent course of investigation.

The majority of the material is presented in the form of drawings with accompanying explanatory notes. The sequence of presentation is such that we begin with those forms of spherical subdivision which are related to the more familiar latitude-longitude or bi-polar system. From these, we progress to the multi-polar systems of spherical symmetry. Finally, we include a description of the mathematical techniques necessary for calculating dimensional properties of the various figures.



A great circle plane is defined by the intersection with the surface of a sphere of a flat plane passing through the center of the sphere. In figure 1, the familiar Bi-polar system of spherical subdivision is shown in which a series of great circle planes are arranged about a common axis. These planes are in turn penetrated by a series of planes passing through the sphere at right angles to the common axis of the great circle planes. Each of the new planes with the exception of the equatorial one describe circles of smaller diameter than the sphere and are known as lesser circle planes.

This type of subdivision naturally gives rise to four sided faces. However, each longitudinal rank of faces must terminate at the poles with one face of three sides. Note as well, that in this system there are always as many faces joining at a pole as there are great circle subdivisions at the equator.

The other portion of this figure describes how we may procede to other types of subdivision based upon this system. The two types of symmetry shown are developed in one case by maintaining the great circle continuities while alternately interrupting those of the lesser circles. In the other case, the lesser circle continuities are sustained while those of the great circles are alternately interrupted. It is seen that hexagonal and triangular patterns arise quite naturally from this system. Moreover, from the point of view of economy of joining, the hexagonal systems are superior to those of the four sided faces. For the hexagons, though equal in area to the four sided faces, have a shorter perimeter and thus represent a considerable saving in total length of joining edge. Note also that in the latter two systems the number of cells joining at any point other than the poles is never greater than three.

Figure 2 shows other developments of the Bi-polar system. In these cases the patterns have been altered to permit the cells to maintain approximately the same relative proportions as they move from equator to poles. This method of subdivision cannot be rigourously carried out since an infinite number of cells would be required before the pole could be reached. It is of some interest that as the equatorial cells are reduced in size, the difference in size between the largest and the adjacent smaller cells is less.

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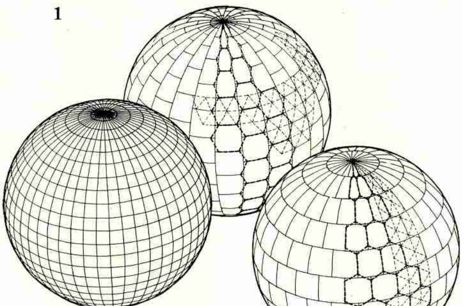

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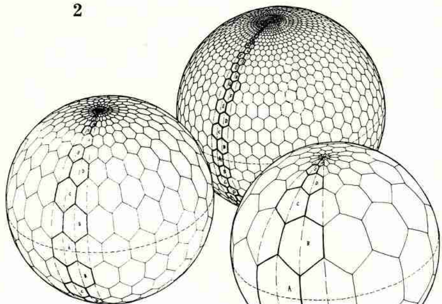

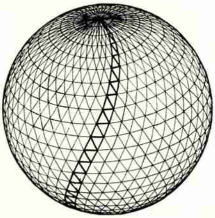

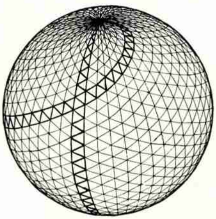

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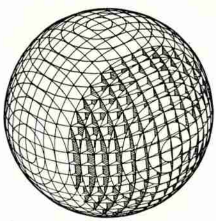

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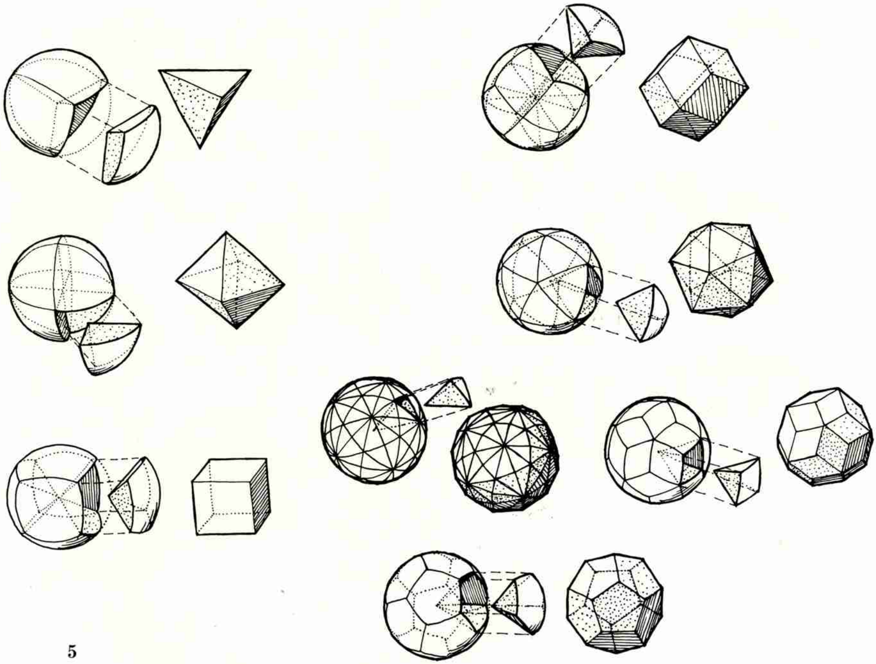
Figure 3 more fully describes the triangular patterns suggested in Figure 1. The illustrations show the resultant patterns when great circle and lesser circle continuities are held. In each case, the continuities established by the sides of the triangles not lying on the continuities of great or lesser circles are analogous to the "rhumb lines" of the mercator map projection. With this type of subdivision, the number of kinds of facial types is twice the number of divisions between the equator and a pole.

Figure 4; Before turning to a more general approach to multi-polar systems, we have included this seemingly simple system of a 6 pole lesser circle method of subdivision. Although conceptually this is the essence of simplicity, or at least it seems so, it leads to an highly complicated series of facets. It appears to be more of an item of curiosity than of utility. Moreover, because its generation is based upon lesser circles its mathematical generation is not amenable to easy calculation by trigonometric techniques - since spherical trigonometry deals with great circle patterns.

Beginning with Figure 5, we turn to a more general approach to spherical subdivision. The previous figures have given us a notion of the properties of spheres subdivided in terms of a single axis of symmetry. This method seems admirably adapted to coordinating points on the surface - for we can define a point anywhere on the surface by a pair of numbers - one of which defines its lattitude or distance from a pole and the other number, its longitude, which defines its angle of rotation with respect to pre-assigned line of longitude. The unit modulus of this system is defined by a spherical isosceles triangle the base of which is the equatorial planar edge, the two equal sides being the longitudinal great circle planes joining the equator with the poles. We see that if we wish to divide our sphere into units smaller than this we must either increase the number of great circle planes - thus increasing the polar congestion - or we must divide up the unit modulus in any one of a number of possible ways. Nevertheless, however we carry out this latter type of division, we cannot escape the fact that we must create a relatively large number of differently shaped parts. It therefore seems intuitively evident that to surmount these difficulties we must do two things; first, we must seek means of reducing the size of the unit modulus in terms of the central angles it subtends. And second, we must find a way to symmetrically distribute our "congestion" to a greater number of points than the two poles of the latitude-longitude system.

To take up the second of the above consideration first, we must look into the ways we may evenly distribute points over the surface of a sphere. If these points are so distributed, we may connect them with portions of great circles and thus divide the surface into identical units. Moreover, if we are in the business of distributing points evenly over a spherical surface - and we intend to connect these points with geodesic lines - are we not then simply constructing spherical versions of planar bound polyhedra? It turns out, of course, that we are. Furthermore, the polyhedra which lend themselves most gracefully to our purpose seem always to possess an even number of vertexes or poles. Though we may construct symmetrical figures with odd numbers of vertexes, they must always arrange themselves about a single axis of symmetry - and they are therefore always related to the latitude-longitude system from which we wish to escape.

Thus, since we wish to escape two poles, we must go to four poles for our first polyhedral subdivision. Figure 5 shows us the way we may distribute points in terms of polyhedra possessing the properties of being made up of faces constructed of unit lengthed edges, uniform facial angles between edges of face, etc. They



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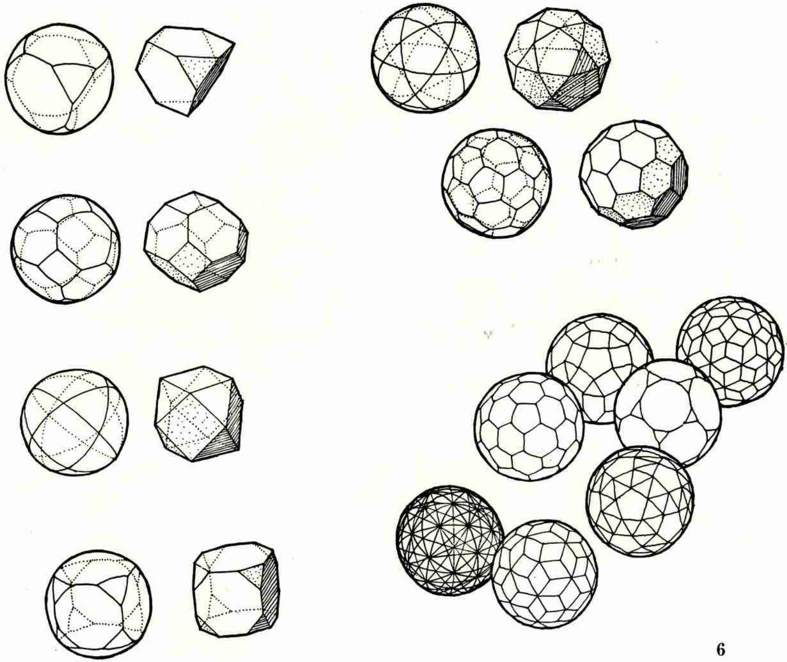
are the so-called "Platonic" solids. We have included the Rhombic versions of these figures as well. The figures are, starting at the upper left, the four poled or vertexed, four faced tetrahedron. Below this is the six vertexed, eight-faced octahedron. Next is the eight vertexed, six faced cube—which also may be considered to be the rhombic development of the tetrahedron—since each of its six faces can be placed symmetrically tangent to the six edges of an inscribed tetrahedron. This is followed at the upper right by the Rhombic dodecahedron, a development of the octahedron and cube. This figure manages to symmetrically distribute the six vertexes of the octahedron and the eight vertexes of the cube, fourteen points in all. Just below this we find the twelve vertexed, twenty faced icosahedron. the twenty vertexed, twelve faced pentic-dodecahedron and the Rhombic version of these last two items—the Rhombic triacontahedron. This



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latter figure symmetrically distributes 32 points over one spherical surface. These points are the vertexes of icosahedron and pentic-dodecahedron. To the left of the Rhombic-triacontahedron we have shown the figure obtained when we draw in the axis of the diamond shaped faces of the triacontahedron. This figure establishes fifteen interlocked great circle planes with sixty-two points or vertexes—moreover, it divides the surface of the sphere into 120 similar triangles which would be identical were it not for the fact that 60 are "right handed" and 60 are "left handed". This seems to be the highest order of unit symmetry we may develop on a spherical surface. If we wish to subdivide farther, we must employ more than one kind of face.

Figure 6 describes the symmetrical divisions obtained when we permit more than one kind of face on our polyhedron—though we still hold to the restriction of a unit length of edge and uniformity of angular condition within a given face in most of the cases shown. Most of these configurations are what we call "archimedian" solids. Beginning at the upper left, we have a vertexially truncated tetrahedron which symmetrically distributes twelve vertexes. Below this, a similarly truncated octahedron which distributes 24 vertexes. Below these we find two versions of similarly truncated cubes. The first of these has been named the "dymaxion" by Mr. Richard Buckminister Fuller and it occupies a prominent position in his geometrical investigations. This prominence stems in part from the fact that the points on its surface represent the points of tangency of equal sized adjacent spheres when they are packed together so as to fill a minimum amount of space. This figure distributes twelve vertexes symmetrically over the spherical surface. The second of these, made up of eight triangular and six octagonal faces establishes 24 points on the sphere.

On the upper right, we see the vertexially truncated icosahedron, (or truncated pentic-dodecahedron) which establishes 30 points on the spherical surface. Below this is a figure made from the twelve pentagons of the dodecahedron plus 20 hexagons which symmetrically distributes 42 vertexes. Below this we see several extensions of the icosahedral-dodecahedral symmetry into higher numbers of symmetrically distributed points. One of these figures seems to be of particular interest and is presently being investigated by the author. This is the figure located approximately in the middle of the cluster at the lower right of the figure. It is composed of twelve pentagons and 80 equilateral triangles, and is the only one of all the figures shown which possess the "right" and "left" property of "spiralness". There is a figure similar in principal to this one composed from 6 squares and 32 equilateral triangles which has not been shown in this article.

Figures 7 through 12 show various methods by which we may further subdivide the "Platonic" solids and their Rhombic extensions. The first of this group, Figure 7, shows one method for developing smaller triangular subdivisions on a spherical tetrahedron. This subdivision is accomplished by radially extending straight line patterns on the surface of the "flat" tetrahedron to the surface of the surrounding sphere. Since great circles lie in flat planes, any extension of a straight line on one of the flat faces of the tetrahedron automatically will lie on a great circle

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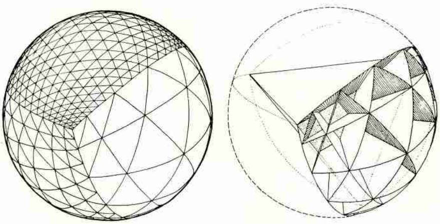
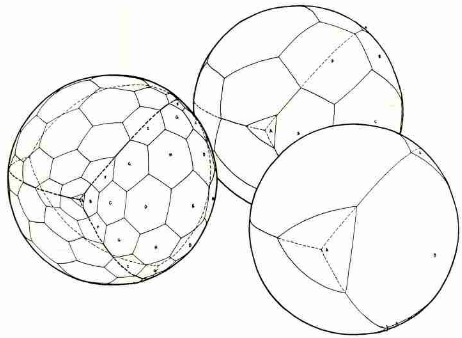




continuity on the surface of the sphere—provided that the straight lines' extension to the spherical surface is radial. It can easily be seen that as the subdivision of the tetrahedronal face increases, the number of different kinds of faces on the spherical surface increase as well. It can also be seen that the size differential between these various facets is a function of their distance from the polyhedral surface. Thus, the smallest facets are to be found in the vicinity of the vertexes of the tetrahedron while the larger facets are found over the centers of the face. In contrast to the bi-polar system, there is no tendency for an inordinate number of faces and edges to come together at a single point.

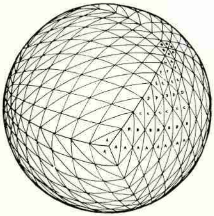
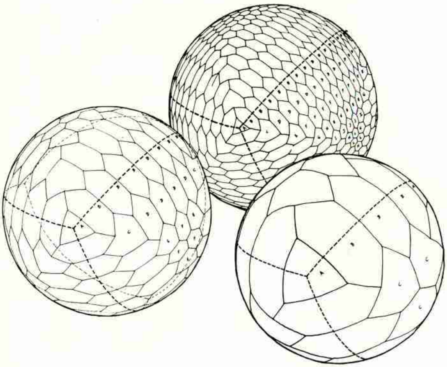


Figure 7 also includes some examples of the derivation of other types of cellular division based upon the triangulated grid. The examples show only hexagonal patterns but it can easily be seen that four-sided patterns could have been derived as easily by suppressing one set of great circles of the triangulated pattern.

Figure 8 shows another way of triangulating the spherical surface starting from a tetrahedronal pattern. The basic modulus of this pattern is an equilateral diamond shaped facet which is developed by passing great circle arcs from the tetrahedron vertexes to the centers of the tetrahedron faces. In the case of the tetrahedron, these resultant diamond facets are actually the edges of a spherical cube. (see figure 5). The great circle arc of the tetrahedron edge, which passes along the diagonal axis of the diamond face, is next subdivided into any even number of divisions desired. Great circle arcs at right angles to the tetrahedron edge are then passed through the points established by the equal subdivisions of the tetrahedron edge and are extended until they touch the edges of the diamond. Two sets of great circle arcs are then passed through these arcs to complete the triangulation in the manner shown in the drawing. It can be seen that the number of kinds of triangles thus developed are equal to the number of divisions originally made along the tetrahedron edge divided by two. The total number of triangles per sphere $(N) = 2d^2D$, where d is equal to the number of kinds of triangles and D is equal to the total number of diamonds on the spherical surface. In contrast to the patterns found in figure 7, notice now that the larger faces are found at the vertexes and over the edges of the tetrahedron face, while the smaller faces are to be found over the centers of the tetrahedron



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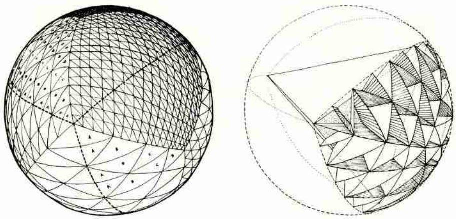
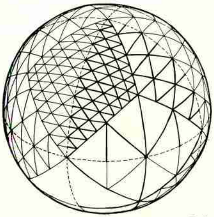
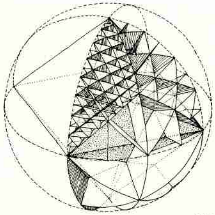
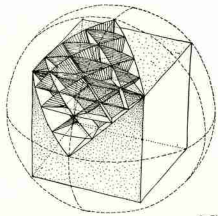




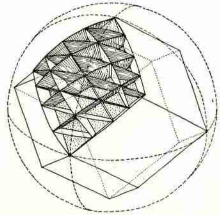
9A



9B



9C



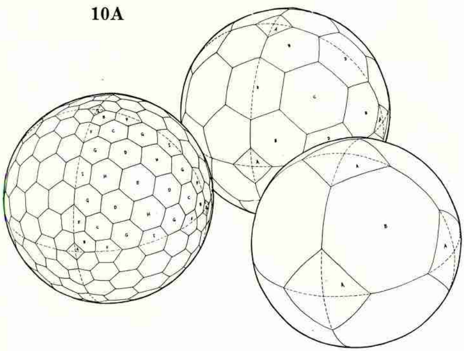
9D

faces. The reason for this occurance is that in this type of grid we are subdividing on the basis of a spherical modulation rather than by establishing our primary modulation on one of the flat faces of the circumscribed polyhedron. It is further pointed out that there is a considerably smaller difference between the largest and smallest faces in this type of subdivision than in the type shown in figure 7. As a consequence of this, edge length differences are smaller, numbers of different edges are less and angular conditions between the faces are greatly reduced in number. As in figure 7, figure 8 includes the development of other type cellular division based upon the triangulated grid.

Figures 9 and 10 show the application of the technique of subdivision developed in Figure 8 to that group of poylhedra related to the octahedron. We see that this grid encompasses the geometrical properties of the octahedron, cube and Rhombic-dodecahedron. Figure 9a shows the triangulated spherical grid and its development with respect to the even subdivision of the octahedron edge. Figure 9b shows the grid and its relationship to the flat faced octahedron. Figure 9c relates the same grid to the cube, and 9d to the Rhombic-dodecahedron.

Figure 10a indicates cellular patterns developed by triangulating the face of an

10A



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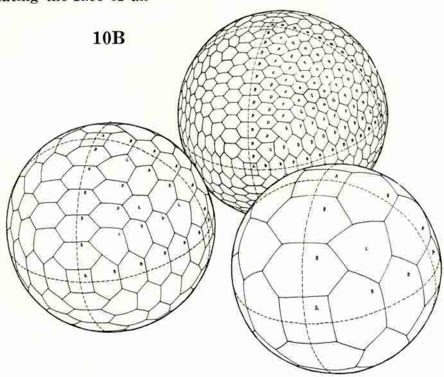
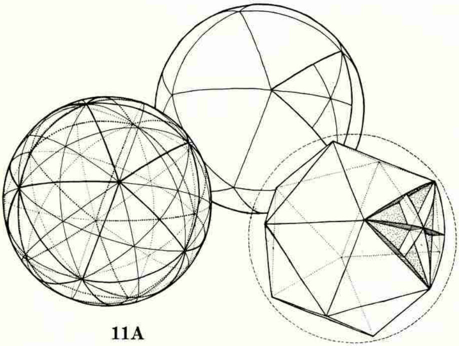
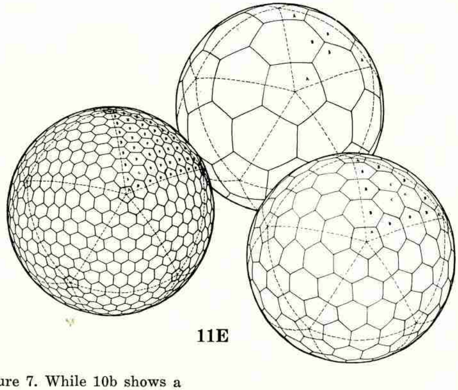




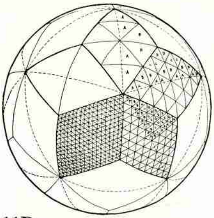
11A



11E

octahedron in a manner analogous to that shown in Figure 7. While 10b shows a similar cellular pattern, but in this case the pattern is based upon the gridding system developed in Figure 9. The remarks made concerning the differences between the grids shown in Figure 7 and 8 apply equally here. Attention is called to one further point, however. Careful inspection of Figures 10a and 10b will show that the sphere with the greatest number of cells in each of these figures are composed from the same number of different kinds of cells (9 in each case); however, the total number of subdivisions is much greater in figure 10b than in the one in 10a. (196 subdivisions in 10a, 580 in 10b.)

Figure 11 deals with subdivisions based upon an icosahedron which has been subdivided in a manner similar to that seen in Figures 9 and 10. Figures 11a through 11d shows successively the triangulation patterns as they relate to the even modulation of the edge of the spherical icosahedron, the polyhedral icosahedron, the pentic-dodecahedron and finally the rhombic-triacontahedron. In each case, the 15 great circle spherical pattern has been included in which the linear emphasis has been placed upon that portion of the grid which relates to the accompanying polyhedral form.



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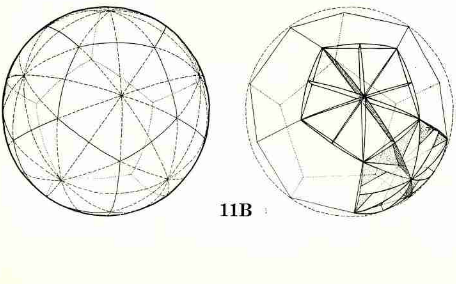

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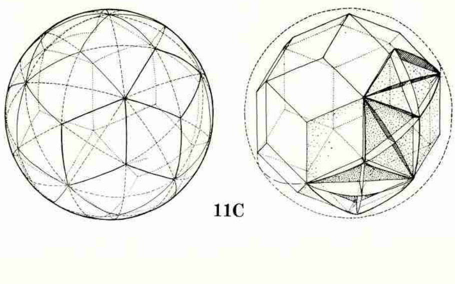

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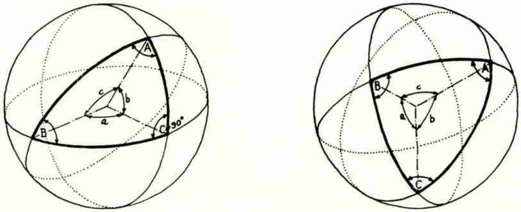

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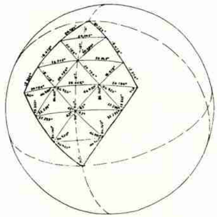

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Figure 11e shows the extraction from the triangular grid of the resultant hexagonal patterns as the edge of icosahedron is increasingly subdivided. It is interesting to compare the sphere with the greatest number of cells here shown with one related to the octahedron (see Figure 10b.). The octahedronal figure has nine different kinds of cells on its surface and the total surace is divided into 580 divisions by them. In the case of the icosahedral figure now under examination, it can be seen that this figure is divided by only six different kinds of cells and the total number of cells on the spherical surface is now 650. Moreover, the difference in size of cell on the icosahedral figure is considerably smaller than on the octahedral one and the resultant edge length and angular differences are correspondingly smaller.

It seems highly probable that so long as we establish the criteria of minimization of facial size differential, edge length differential and angular differential, the icosahedral family will have to serve as the polyhedral basis for the subdivision. For if we try to go beyond this point, we are faced with the necessity of employing some one of the figures shown in figure 6, the so-called "Archimedean" solids—and when we do so we are faced with bi-facial symmetry and a resultant complication in our number of moduli. There are certain special cases where advantages accrue to systems based upon these forms but, in general, "icosahedronness" seems to be the apogee of this series.

Since the magazine publishing this piece is not large, we are forced to abandon any notion of going into greater detail in our discussion. However, Figures 12 and 13 have been included to enable those who are interested in further study to carry out with a greater degree of precision some analysis of the possibilities presented. Figure 12 is simply an exposition of spherical trigonometry presented in a way that the author considers effective for the accomplishment of this task. The diagrams and equations seem to be self explanatory. Figure 13 shows an example of a grid worked out in the manner of Figure 9. The numerical indications at the vertexial corners indicate the spherical surface angles while the angular data along the edges of the faces indicate the central angles subtended by the edges.

The Right Spherical Triangel

(C = rt. angle)

The Cases

I. Given A, B, C, determine a, b, c.

$$\cos a = \frac{\cos A}{\sin B}$$

$$\cos b = \frac{\cos B}{\sin A}$$

$$\cos c = \cos a \cos b$$

II. Given A, C, c, determine a, B, b.

Determine a by Law of Sines

$$\sin b = \tan a \cot A$$

$$\cos B = \tan b \cot c$$

III. Given A, C, b, determine a, c, B.

$$\cos B = \sin A \cos b$$

$$\cos a = \frac{\cos A}{\sin B}$$

$$\cos c = \cos a \cos b$$

IV. Given C, c, a, determine A, B, b.

Determine A by Law of Sines

$$\sin B = \frac{\cos A}{\cos a}$$

$$\cos b = \frac{\cos B}{\sin A}$$

V. Given a, b, C, determine A, B, c.

Determine A and B by Law of Sines and

$$\cos c = \cos a \cos b$$

The General Spherical Triangle:

Sine Law:

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

The Cosine Law for Sides:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

The Cosine Law for Engles:

$$\cos A = \cos B \cos C + \sin B \sin C \cos a$$

$$\cos B = \cos A \cos C + \sin A \sin C \cos b$$

$$\cos C = \cos A \cos B + \sin a \sin B \cos c$$

The Cases.

I. Given 2 sides and the included angle:

Find other side by Law of Cosine for sides.

Find other two angles by Law of Sines:

or use

$$\frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} = \frac{\tan \frac{1}{2}(A-B)}{\cot \frac{1}{2} C}$$

$$\frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} = \frac{\tan \frac{1}{2}(A+B)}{\cot \frac{1}{2} C}$$

Note: by adding and substracting we obtain B and C.

Get other side by Cos Law for angles.

II. Given 2 angles and included side:

Given A, B, c, to determine a, b, C.

Determine C from Cos Law for angles.

Determine a & b from Sine Law.

III. Given 2 angles and side opposite one angle:

Given A, B, b, determine a, c, C.

Determine C by second method shown in Case I.

Determine c by Law of Sines.

Note: Although there are two other possible cases, (i.e. Given A, B, C, a, b, c,) they do not apply in these cases for at least one surface angle or one central angle is always given at the outset, or, the problem is more readily analyzable as a right spherical triangular case.

Felix J. Samuely

SKIN STRUCTURES

THEIR RECENT APPLICATION IN GREAT BRITAIN

Felix J. Samuely well known English structural engineer currently teaching at the Architectural Association, London.

During recent years, shell construction has been used more and more for roofs, and has become increasingly familiar. The familiar shell, however, is either the standard cylindrical or spherical shape, and it is sometimes forgotten that roofs of this type need not necessarily be curved, but can be made up out of straight lines. Once this point is appreciated, the field immediately widens, and many different roof outlines become possible. Such roofs are known by several different names, for instance, folded slabs, hipped plate roofs, etc., but they can all be grouped as skin structures.

Generally speaking, a skin structure could be defined as a structure in which the stiffness of the skin is used as the basis of the construction. The stiffness of a slab in its own plane is considerable, and cannot be better demonstrated than by the classic example of laying a thin piece of cardboard between two supports. This cardboard is quite incapable of withstanding loads, but once the cardboard is turned up on its side, it can carry a load applied to its edge. Two pieces of cardboard, inclined and resting against one another at the top, are capable of carrying quite a considerable load. See Figure 1.

The unsupported ridge in the center can carry a vertical load by resolving it into two loads, one in the direction of each piece of cardboard. The surface of the cardboard has become, in effect, a skin structure. The ends of the cardboard should be held together against each other to prevent sideways movement.

This principle is still valid if the cardboard is replaced by solid concrete or by latticed steelwork.

It can be seen from this that the placing of structural member makes a tremendous difference to its carrying capacity, and the art of skin structures is to recognize the position in which a member has its greatest strength, and then to arrange it accordingly. As members are strongest in the plane of their skin, forces occurring at right angles to them must be resolved into components in the direction of the skin.

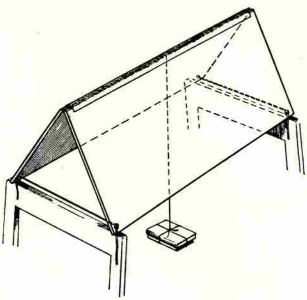
The stiffness of a roof construction has always been taken into account for distributing wind forces. A solid roof is used to distribute these forces to cross walls or wind frames, and if the roof is not solid, that is, consists of sheet material that is not strong enough to withstand forces even in its own plane, bracing is introduced to bring the wind forces to the walls or frames, and this bracing is always introduced in the plane of the roof.

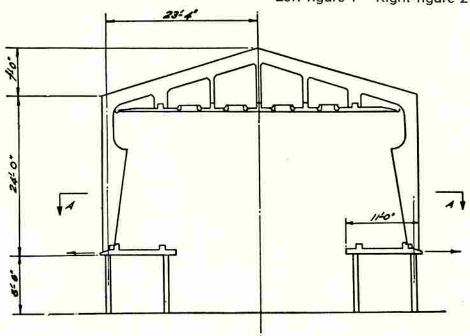
This is common practice because it is quite easy to visualise wind forces acting in the plane of the roof. What is done for wind, however, can quite easily be applied when transmitting the thrust of intermediate frames or arches to cross walls or cross frames.

An example of this is shown in Figure 2, which gives the cross section of a church at Poplar, in East London. The thrust at the bottom of the rigid frames would not be taken separately for every frame, and the slab is used for bracing. In this particular case the frames started only at first floor level, so that it is the slab at first floor that is used to transmit the horizontal forces to the end.

A four-hinged frame can also be stabilized by the stiffness of a horizontal slab in its own plane. This horizontal slab, in its turn, can carry the loads to cross frames, cross walls or gable walls. Four-hinged frames have the advantage that the hinges can usually be positioned at the most economical points, and this permits of easy erection for precast concrete or steelwork. It must not be forgot en, however, that the frame on its own is unstable, and only becomes stable after the s iffening slab is in place. With steelwork, for instance, the frame must either be guyed temporarily, or braced in the plane of the slab. In spite of its advantages, the four-hinged frame is very rarely used in practice. Engineers and architects, not to mention local authorities, are always reluctant to use a sys-

Left figure 1 Right figure 2





tem which is not complete in its own plane, but relies for its stability on the combined action of at least two planes; without a certain mental alertness, the possibilities of such constructions are easily overlooked.

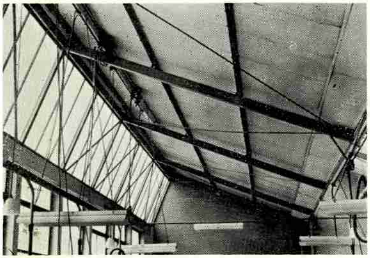
The fact that stiffness can be provided by latticed construction at well as solid construction is extremely important. It means that skin structures can consist of latticed steelwork or precast concrete, as well as of in-situ concrete.

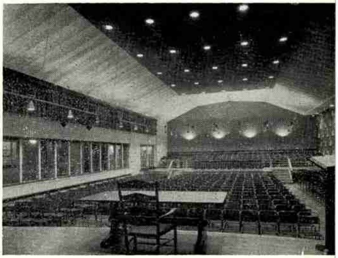
The main application for skin structures is still, of course, to roof construction, and although they can be used for any roof that has more than one plane, the first to be considered is the simple pitched roof. There is, however, one point which must not be overlooked. When long span slabs are used on edge, there is always the danger of excessive shear stresses. In practice, shear is not usually a serious matter when the ratio of length to width of any slab is more than 10. Where the width of a slab is greater, it might be necessary to thicken the slab beyond what is necessary to take the bending stresses as well as to reinforce it. This additional shear reinforcement may often detract considerably from the economy, particularly when compared with steelwork.

With a single pitched roof, the span of the roof slab is from the ridge to the eaves, that is half the actual span of the whole roof. Instead of spanning the slab itself from the eaves to the ridge, it is quite possible to span beams this way, and span the slab between these beams. Latticed construction can be used in place of a solid slab, and Figure 3 shows a factory at Stockton on Tees where this was done. In this building there are cross frames every 85′ 6″ and a latticed girder comprising four panels is arranged in the sloping roof to take the component forces of the three intermediate ribs, simply spanning from eaves to ridge, which in their turn, carry the purlins. While the purlins span only 21′ 4½″ the latticing, which does not use much material, makes it possible for the final span to be increased fourfold.

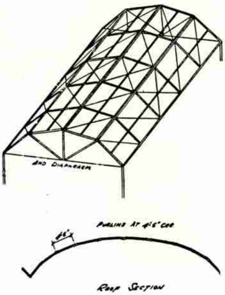
Figure 4 shows the application of this type of construction to a northlight roof. With a northlight roof it is usually possible to fabricate the steep latticed girder and transport it in one piece to the site, but the girder in the other plane is usually too wide for transport as a whole. A practical method of erection is, therefore, first to erect the northlight truss, stiffen this by means of the verticals of the other truss, and insert the diagonals afterwards. Unless the chords of the two trusses which come together consist of a common member, they must be joined together in such a way that they can act together. This might present difficulties at the eaves if gutters have to be accommodated, as the chords have to be spaced apart the width of the gutter, and it may be necessary to introduce horizontal bracing across the gutter.

As the gutter in this case becomes an important feature, its dimensions should be decided upon before the details of the roof are completed.





Below figure 3. Above, left to right figures 4, 5 and 6



It is often required that glazing should be arranged in front of a latticed girder, and generally the diagonals behind the glazing do not obstruct the light to any appreciable extent. To keep the diagonals as unobstructive as possible, round bars can be used.

The systems described so far referred to simple pitched roofs. The same principle can be applied to other shapes. Where two planes come together at a ridge, the intersection acts as a beam, although there is actually no beam there. With the simple pitched roof, by considering this intersection as a beam, the span of the slab was halved. Where even this half span is considerable, it might be advantageous to break the span up into smaller parts. Slabs of about 6′ to 8′ are usually the most economical, as this allows the thickness of the slab to be reduced to a minimum, say $2\frac{1}{2}''$ to $3''$. Each fold in the slab acts as a beam and the more folds there are, the smaller become the local bending moments owing to the load being transmitted to the folds of the slab.

This type of construction has been developed more in reinforced concrete than in steelwork, and further examples are given in figures 5 and 6.

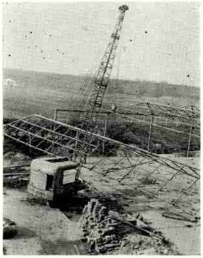


Figure 5 shows the roof over a brewery at Alton, England. The sloping planes are almost completely glazed. Architects often ask for Virendeel girders to be used behind the glazing believing that in this way the maximum amount of light is obtained. This is a mistaken belief as, owing to the very large local bending moments in such girders, all the members become excessively heavy thus actually reducing the glazing area. A simple latticed construction behind the glazing usually takes away much less light. The construction shown in Figure 5 is often the most economical, in spite of the depth of the beam at the bottom of the sloping plane. A certain thickness must be left here in any case for waterproofing, and as the light near the gutter is the least effective, the increase in beam depth is not important. The strip above the beam can then be left completely free of obstructions.

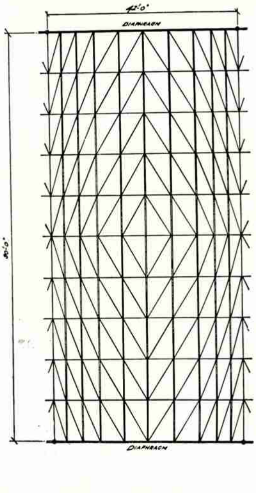


Figure 6 shows a concrete roof where precast units were used for the lower part of the slab in order to save shuttering (ed. note: formwork). In this case the precast units were trough-shaped, which makes them stiffer than flat slabs. The units were laid side by side, in four rows, and were temporarily supported on props along the folds of the roof. The troughs were strong enough to carry their own weight and that of the wet concrete to be poured on top to complete the roof, so that no intermediate strutting was necessary. The topping concrete was only 1½" thick, making a total thickness of the shell of 2½"; however, the stiffness of the shell was equal to that of a 6" slab, because the depth of the coffering, formed by the troughs, was the measure of the rigidity. This great rigidity, achieved with a small thickness of concrete, made the construction extremely economical. This particular roof, which was for a school assembly hall at Wigan in Lancashire, was 50′ across and approximately 90′ long. Figure 6 shows the inside of the completed hall.

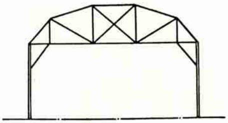
Again, such roofs can be carried out in latticed steelwork. The distance between purlins in steelwork is usually dictated by the maximum span of the roof decking. By having several folds in the roof, the chords of the trusses can act as purlins, with the roof decking spanning directly on to them. This saves intermediate purlins, which would tend to introduce bending moments in the verticals of the latticed girders. As in concrete, the intersection or fold between two latticed girders can be considered as a beam because any forces that act on this line can be resolved into components in the two planes of the latticed girders, and these components can be taken by the girders.

Figures 7 and 8 show two examples of laticed steel skin roofs, the first a roof with a column grid of 60′ x 30′ and the second one of 90′ by 42′. In both cases the latticed girders were designed in tubular steelwork because of the greater simplicity of the connections.

These girders can either be erected on a scaffolding, or alternatively one whole panel consisting of several lattice girders can be assembled on the ground and lifted up into position in one piece. With this latter method, however, sufficient repetition must be provided to make the use of special lifting apparatus economical.

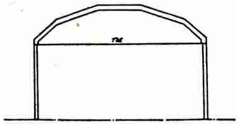
With all skin structures, whether in concrete or in steelwork, the slabs or girders in the planes of the roof transmit forces along their own planes to points of support at the ends. These points of support must be capable of collecting up the forces and transmitting them to the ground.

There are several different methods of collecting these forces, but it must be remem-

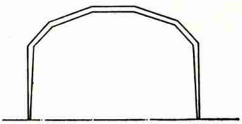


LATTICED TRUSS
(a)

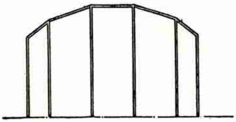
SOLID BEAM
(b)



RIGID ARCH WITH TIE
(C)



RIGID FRAME
(d)



COLUMNS FOR EACH FOLD
(e)

bered that at each supporting point vertical as well as horizontal forces can occur. In theory, it would be the simplest to have a column under each fold, the columns rigidly fixed to the foundations, so that they can take horizontal as well as vertical loads. This can be cumbersome and result in too many columns and expensive foundations.

Another possibility is to have a rigid frame following the outline of the roof, and strong enough to transmit all forces.

The third method, and that most frequently adopted, is the diaphragm. For a simple pitched roof, such a diaphragm may consist of a single tie member, which would triangulate the system and bring the load to the columns at either end of the diaphragm. Where there are more than two planes to the roof, the diaphragm must be latticed, sufficiently triangulated to hold each point in position. Or latticed constructions can be replaced by solid diaphragms. Figure 9 shows a number of possible diaphragms. It is very important to appreciate that a skin structure acts somewhat like a large beam spanning between the diaphragms.

With all skin structures there must, therefore, be at least two diaphragms or lines of supporting columns, and the most logical place for these is in the gable walls. If a building is too long for just two, intermediate diaphragms, together with their supporting columns, can be introduced.

One important point to be considered with this type of roof is the long edge (usually at eaves level). It has been stated that the intersection of two slabs, at an angle, can be considered to act as a beam, but of course, along the edge of the building there is only one slab. This edge usually coincides with the outside wall of the building and, in this case, it is a simple matter to support this edge directly on the wall, or on columns, with a comparatively shallow beam spanning between them. If one or both sides of a building are to be left open, the last roof slab must cantilever out from the rest of the roof. With this arrangement it is better to keep the outer slab as small as possible. There is no difficulty in cantilevering a concrete slab, but with latticed steelwork the vertical members of the girder would have to be rigidly connected to the adjoining girder to transmit the cantilever bending moment.

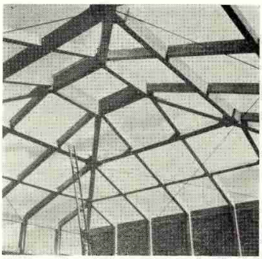
With factory buildings, it is often necessary to support extra loads on the roof construction, e.g., cranes, gantries, etc. With traditional construction, these are suspended from the roof trusses, or from special beams spanning between the roof trusses. The same arrangement might be kept to with skin construction, by spanning a gantry or crane from diaphragm to diaphragm, provided these are not too far apart. If a gantry or crane runs across the folds, it can be supported from each fold, thus reducing its span. If it runs parallel with the folds, it is better to arrange it immediately under a fold, and there will then be no difficulty in supporting it at intervals along the fold. The beam along any fold is quite capable of taking this additional load and of resolving it into components in the planes of the roof.

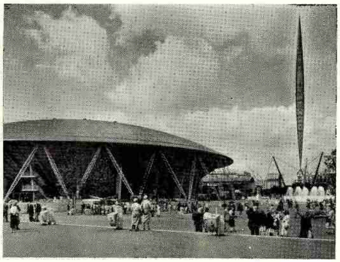
Skin construction need not be restricted to ordinary pitched roofs, or a series of pitched roofs. They can be hipped at the ends, or can slope in four or more directions. This can be done in solid concrete (and if there are a large number of small planes this would gradually be coming back more to the standard shell construction) or in latticed construction, either precast concrete or steelwork.

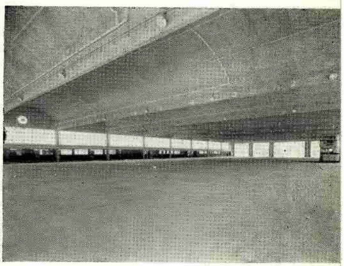
Figure 10 shows an example of latticed work, with precast concrete compression members and round steel bars for the tensile members. This roof is on a square base 40 ft. x 40 ft., but the type of construction could be used for larger buildings or for rectangular ones, and the material changed to either steel or timber.

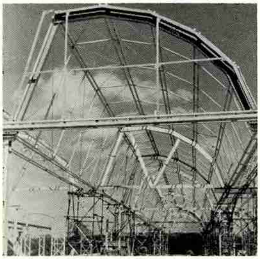
When the number of folds in a skin roof is greatly increased, so that the span of the planes becomes smaller and smaller, the result is what is commonly called shell construction. The distance between the folds becomes so small that the roof outline is a continuous curve. This type of construction is the most frequently used, and lends itself more to reinforced concrete, the medium for which it was originally designed. That it is possible to use latticed construction however, is proved by the famous hangar near Rome, designed by Nervi, which was a diagrid construction of precast concrete, the members having a straight axis but a curved upper surface. Provided that light covering materials can be found which are suitable for curved surfaces, it should be possible to develop this system, both in precast concrete and in steelwork, or timber and aluminum, to produce lighter shells than shell concrete. The Dome of Discovery on the Southbank for the 1951 Exhibition was constructed on this principle, but in aluminum, see figure 11.

Left to Right
Figures 10, 11 and 12








Left to right Figures 13 ,14 and 15

With an ordinary shell construction, the local bending is reduced to a minimum, but against this advantage must be set the cost of the curved shuttering. Only if the shuttering can be used for several buildings, or if the repetition within one building is extensive, can this disadvantage be ignored. An ellipse is the ideal shape for a shell roof, but here the shuttering would be even more difficult. Another type, which is easier to shutter, and is becoming very well known, is one where the ellipse is replaced by a segmental arch; the vertical portion, which resembles a beam, is, in fact, the tension part of the shell. See Figure 12. Many constructions of this type have been carried out, and very frequently the shell is constructed across the longitudinal axis of the building so that a better repetition is obtained. This is also shown in Figure 12.

Various shells of this type have been used for northlight roofs and for cantilevers, and in Figure 13 can be seen a case where the diaphragm is placed on top, and the two halves of the roof cantilever out on either side. As this is for the roof over a railway station platform, it was essential to keep the outer edge free of columns or supporting walls.

A similar construction in steelwork can be seen in Figure 14. This is a northlight roof for a factory in a new town south of London. It consists of a number of tubular latticed girders side by side forming the surface of a shell northlight roof, very similar to the more standard types of concrete shells. The construction is covered by insulation board and curved cement asbestos sheeting. While the outer appearance is that of a curve, the cross section is, in fact, a series of straight lines, each at a slight angle to the next one. The loads occurring at the folds are resolved, as usual, into the components in the plane of the roofs.

As already mentioned, a useful application of shell construction is the dome. An example of this, in addition to the Dome of Discovery seen in Figure 11, is the rubber factory at Brynmawr shown in Figure 15. A dome on a square base is a problem of its own, and at Brynmawr, as can be seen, segmental spandrels were used.

It can be seen from some of these examples that steelwork can be used for skin structures just as easily as concrete. Particularly for industrial buildings, steelwork, covered with light roofing material, is often to be preferred, as alterations are easier, and in industry, alterations must always be considered. On the other hand however, the use of precast concrete as permanent shuttering makes concrete cheaper than steelwork, but it is too early yet to lay down definite comparisons.

One point must always be kept in mind. It was mentioned earlier that shear stresses are of vital importance. Reinforced concrete, unfortunately, is particularly bad for taking shear, so that with shell concrete the shear stresses have to be taken entirely by steel re-enforcement. For heavy shear, the amount of reinforcement required for this purpose might be greater than the total tonnage required for a much lighter latticed steel roof. The position has been improved considerably by the introduction of prestressing as this produces compression stresses which reduce the diagonal tension over part of the area.

If the development of latticed steelwork for skin structures continues its present trend, I think it can safely be predicted that ordinary reinforced concrete may be used only for shells of up to about 30′, and steelwork and prestressed concrete for the larger ones. Whether the shells will be curved or a series of straight planes, depends on economy and architectural requirements, but as has been shown by the preceding examples, the architect has a wide field from which to select his roof profile, and he can be confident that concrete or steelwork, curved or straight, the engineer can meet his needs.

Benjamin B. Taylor

visitors to the school of design for the spring semester:

R. BUCKMINSTER FULLER — *(Month of February) Designer conducting special problem with a group of architectural students.*

RANSOM R. PATRICK — *Head of the Department of Art, Music and Aesthetics at Duke University, Durham, North Carolina, conducting a course in the Philosophy of Design with with fifth year students.*

BRIAN HACKETT — *(February 20 to March 20) Senior Lecturer in Landscape Architecture and lecturer on town planning at the University of Durham, England, acting as visiting critic on special problem with advanced students in Landscape Architecture and Architecture.*

MARCEL BREUER — *(March 28, 29, 30) Architect from New York conducting seminars on his work.*

MARIO SALVADORI — *(April 4, 5, 6 and May 4, 5, 6, 7) Professor of Civil Engineering at Columbia University, New York, conducting special problem in thin shell structures with fifth year class.*

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Anthony Lord	Asheville, N. C.
Lyles, Bissett, Carlisle and Wolff	Columbia, S. C.
George Matsumoto	Raleigh, N. C.
Lewis Mumford	Amenia, New York
Robert W. McLaughlin	Princeton, N. J.
A. G. Odell, Jr.	Charlotte, N. C.
Page and Smith	Raleigh, N. C.
George W. Qualls	Philadelphia, Pa.
Sam Rosenberg	Raleigh, N. C.
Schnedl and Schnedl	Reidsville, N. C.
G. Milton Small	Raleigh, N. C.
F. Vernon Smith, Jr.	Charlotte, N. C.
Southern Architect	Charlotte, N. C.
Harry J. Spies	Cranford, N. J.
Glenn Stanton	Portland, Oregon
Walter Dorwin Teague	New York, N. Y.
Edwin G. Thurlow	Raleigh, N. C.
Russell S. Walcott	Tryon, N. C.
Ralph Walker	New York, N. Y.
Edward W. Waugh	Raleigh, N. C.
Williams, Coile and Blanchard	Newport News, Virginia
F. Carter Williams	Raleigh, N. C.
Mr. and Mrs. C. H. Winecoff	Winston-Salem, N. C.